

# New challenges, new approaches: A new way to teach Mathematics in Engineering

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**Abstract** - In this paper some ideas, involving methodological changes and mathematical contents for engineering learning, are analysed. The starting point will be the European Higher Education Area (EHEA) which includes a completely new structure of university studies. EHEA will demand, among other things, the production of new didactic materials and the use of the new technologies like an essential educational tool. In this paper we will develop two European projects. These projects may play an important role in the new teaching scenario.

*Key Words:* b-learning, e-learning, Computer Algebra.

## 1. INTRODUCTION

The tailoring of the structure of different university studies to the European Higher Education Area (EHEA), an objective forthcoming from the Bologna Declaration, is the major task outstanding in Europe's different university systems. The reform affects not only the structure of university studies, but furthermore leads to European-wide reflection on the suitability of the syllabuses in mathematics subjects and, of greater importance, on the manner in which mathematics is taught. It is now no longer possible to uphold the same approach to teaching mathematics as fifty years ago and classrooms should reflect the technological revolution that has occurred in recent decades.

Taking into account this new frame in which we must develop our teaching activity, it is necessary to reconsider the role of mathematics in engineering schools. It is evident that mathematics will continue fulfilling a double objective in the new university frame. In one hand they will continue being a powerful formative tool and on the other hand they will continue being the support and background of other academic disciplines. Therefore a mathematical basic background will be essential in the new frame of acquisition of competencies. And the methodological changes affect, in an essential way, to the mathematical topics. Let's not forget that, at least in Spain, we will move from an eminently face to face teaching to a mixed teaching, in which the student, under the supervision of the instructor, must acquire new knowledge across his/her own activity. With this panorama, it is necessary to highlight the importance of e-learning and b-learning techniques in the new academic configuration.

Initially the situation is complicated since inertia exists among some mathematics teachers. They try to avoid the deep change inherent to the new system, and prefer to continue with the traditional system of education. In our opinion, it will not be possible.

In this article, we shall present new tools, forthcoming from the European projects **Building a European database of mathematical e-learning modules (dMath)** [6] and **European Virtual Laboratory of Mathematics (EVLN)** [4], within the Leonardo da Vinci Programme, which will undoubtedly be an extremely useful tool in favour of the content and methodology review that European society requires. We are convinced that only with the collaborative work between mathematical communities is possible to assume the change that is arriving. Former experiences in this area can be shown in [1, 2]

## 2. THE EUROPEAN HIGHER EDUCATION AREA: A NEW TEACHING SCENARIO

In order to find a global understanding of this new paradigm of European university teaching means, we must first outline certain features of the European area that are currently being designed [5, 9]:

- A competency-based teaching offer. Then we can plan and select the mathematical contents for specific students, engineers for example.
- A diversified teaching offer (theoretical and practical teaching, supervised academic activities, independent individual work and so on).
- Concern for the student's overall work, as opposed to the current system in which the only measurement in certain European countries, such as Spain, is the lecture's hours given in the classroom. This work should have a maximum annual volume estimated in 60 European credits (European Credit Transfer System, ECTS). An initial estimate suggests that one ECTS credit is equal to 25-30 hours of student work, including class attendance, laboratories, workshops, individual and group tutorials, individual or group work and assessments.
- Prevalence of student's learning over the lectures provided by teachers.
- Duration of studies that is more adapted to reality.

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The demands on the teaching staff, stemming from the European-wide policy of harmonising higher education studies, are basically as follows:

- Teachers should teach things with added value in the labour market (competencies).
- Teachers should adopt a different approach to teaching (methodological innovation).
- The basic criteria for the planning and development of teaching should be students and their needs (students as a referent).

All this involves a greater dedication from teachers to a student's learning process, which should include the teaching hours, planning, material preparation, suggested lines of work, guidance and supervision and, in general, overseeing the student's entire learning process.

This whole situation requires teachers to take special care when planning the following aspects:

1. The competencies that students will acquire individually within the context of the total offer of competencies that are taught in the subject in question.

2. The academic scenarios in which the teacher wants the learning to take place, referring to specialized classrooms (computer rooms, laboratories, audiovisual rooms, etc.), to libraries and other documentation centres, provided that the students can proceed independently within them. In addition, the proper use of e-learning can contribute to the learning process, taking into account:

- The working process to be followed by the student for the acquisition of these competencies.
- The system of tutoring-supervision that the teacher has adopted to control the acquisition of competencies on the part of students.

3. The system for assessing the competencies acquired by the student on an independent basis. Assessment criteria should be put in place accordingly (results expected in each stage and overall results of the process) and the evidence regarding each criteria (data collected, generated reports, etc.) liable to assessment, the assessment tools and the assessment dates.

The teacher will be required to provide students with material, give lectures, set supervised work or problems, arrange different kinds of tests, use the latest technologies to a greater or lesser extent, etc., in order to generate the number of working hours corresponding to the credits assigned, which are furthermore to be controlled in an effective manner [7].

#### **4. WHAT TO DO WITH MATHEMATICS?**

In our opinion, any University curriculum should be governed by the following two principles:

1. The University must guarantee a supply to its students of the knowledge that will be demanded of them upon their integration into the job market. Consequently, University programs must adapt themselves to the current needs of society.

2. The training of University students must be scientific and students must receive knowledge in advance of what society will demand of them from a purely formal point of view.

Accordingly, mathematics will continue to be necessary within the basic training of the different types of engineer that European society will need in the future.

It should first be noted that the teaching of mathematics in engineering schools should allow students to acquire sufficient skills to be able to analyse and encounter the best solutions to a given problem as a function of the context within which they will be moving in their later professional lives. The ability to find the "best" solution of a given problem necessarily passes through being able to analyse all the issues involved in a given phenomenon, physical or economic, such that the acquisition of this skill will necessarily involve the use of mathematics as a practical tool and as part of the way in which students are taught to think about such problems.

Before addressing the change in contents and the methodological change, we must ask ourselves again about the role to be played by mathematics in the training of engineers.

The principles referred to above mark the route to be followed in University teaching, particularly in the field of mathematics. Thus, mathematics should not be isolated from the rest of the sciences but should form an integrating system of several different systems. Only if this aim is achieved it will be possible to consolidate the mission of the University in society in general; that is, as an innovator of knowledge, thereby preventing scientific and technical stagnation.

To achieve this objective the following guidelines should be followed:

i) The level of knowledge received through University teaching should allow students to adopt a critical and creative stance when they face with a given problem; it should offer them the necessary reasoning tools (skills) for the solution to such problems. A narrowness of view in the transmission of knowledge could become a severe disease in University teaching..

ii) Learning the answer to a problem does not provide an intelligent idea of the process of solution since some step will always be overlooked and this will lead to the generation of several questions. The student identifies this step when reflecting on how the solution to the problem is arrived at. Thus, teaching should promote student's criteria, should foster their ability to reason, and should lead them to be able to handle different theories and methods, motivating them to accept or reject them on the basis of such reasoning.

iii) Scientific advances demand of students the capacity to recycle their knowledge as time progresses and at the same time demand a solid basic training. This should be an objective to be sought. This very dynamic scheme of continuing education means that students must have a sound training in their respective disciplines. Thus, computers may be of great help in this recycling process.

iv) There is an urgent need to provide teaching with a suitable framework of knowledge, since new contributions to overall human knowledge grow daily and their applications are diverse and complex.

Accordingly, instructors must be in possession of an overall understanding of the subject and of the potential applications of the material taught. With this, it would be possible to make a complete program that would gather

together training and compilation, thus imparting the content of the syllabus selectively and with conceptual depth.

This implies the following:

i) The instructor must teach mathematics bearing in mind that for his/her students mathematical subjects are a means, but not an end, to their studies. In sum, mathematics is a **service subject**.

ii) In maths subjects, a field survey should first be carried out to know which parts of mathematics are required and then these should be transmitted in a training-informative way.

iii) A different type of mathematics should be taught for each science, since their demands differ. Even the same topics should be explained and discussed in different ways, depending on the audience that receives them (this is not usually the case in current study plans since, for example, “almost” the same mathematics are taught in all engineering studies, without differentiating their particularities). A **functional** edifice should be constructed, with solid mathematical bases.

Without a good teaching strategy, students may arrive at the mistaken idea that mathematics has been introduced in their studies as a kind of “barrier” of no ulterior use. In general, this strategy involves teaching how to analyse a given situation in abstract terms; that is, modelling a situation. For modelling, three aspects are crucial:

1. Formulation of the problem in mathematical terms. To reach this formulation it is necessary to have an in-depth knowledge of the physical or economic stratum underlying the problem to be solved. In this phase, the mathematical model best suitable to the problem is chosen; that is the appropriate variables are taken and the mathematical problem to be solved is posed. One of the difficulties encountered on passing from the problem to the appropriate language is the need to carry out simplifications of the original problem and here is where it is necessary to discriminate with certainty the magnitudes that may be neglected and those that must be taken into account.

2. Solution of the mathematical model. In this step, the application of mathematics is considered in the most direct sense, together with the theory that will allow that application to be implemented correctly. Then the most appropriate technique known must be applied to solve the problem in question, often with numerical methods.

3. Interpretation and analysis of obtained results. To lend sense to everything that has been carried out, the results should be interpreted through knowledge of the physical or economic problem it is wished to model, contrasting to see whether the results obtained are compatible with the model and with the hypotheses made about it. If the results are not in agreement, the model must be restated.

Seeing the characteristics of the EHEA and the aforementioned demands on teachers, certain changes, sometimes of a drastic nature, are required in our traditional approach to teaching. The changes required do not only affect syllabuses, stemming from weaker initial grounding, but in general affect the knowledge of mathematics acquired in the pre-university stages. This is highlighted by different European-wide reports, such as the PISA report, and there is

also a need to embrace a methodological change that will allow the addressing of students’ new learning needs. The use of the entire potential provided by new technologies applied to teaching may lead to reconsideration of the organisation of different teaching groups, adapting their size to the possibilities of the different computer rooms [10].

Finally, we should change the assessment process, seeking mechanisms that assess the whole learning process. New technologies are once again the key instrument, allowing the introduction of well-designed self-assessment processes, guided practical sessions, etc.

To implement all this methodological changes it becomes indispensable to produce new didactic material to be used by the students in different situations of learning. New material is not reduced to upload the traditional materials and nice power-point presentations on the net. We have to include electronic material, to promote the use of Computer Algebra Systems, on line tutorials, etc. This material will be used according the student’s personal needs, it must enhance the mathematical competences and to promote the collaborative work.

In this frame, we will expose the main characteristics of two new European projects that affect in the creation of new didactic materials, providing new tools in the sense mentioned previously. Both projects are focused on the production of new didactic materials to cover the formative needs of our students in the frame of EHEA.

## 5. THE DMATH PROJECT

A database containing different mathematical modules for the e-learning education has been the main goal of the DMath project [3,6], financed by Leonardo da Vinci program for the period 2000-2006 with the number 2003-N/03/B/PP-165.011. This project, already finished, has been coordinated by the Buskerud University Collage (HIBU, Norway) and in its development the following academic institutions and companies have taken part: Industriell Dokumentasjon (Norway), Soft4Science (Germany), Czech Technical University (Czech Republic), Sogndal College (Norway), Slovak Technical University (Slovakia), University Pontificia Comillas (Spain) and Savonia Poytechnic (Finland). Salamanca University has been the external evaluator of the project.

The developed system contains a XML/MathML editor (Sciwriter is the commercial name), the database or data repository (with the working title “SciLAS System”) that is made up from a number of independent modules, each of them containing different files (also known as Reusable Learning Objects), that can be downloaded individually by users of the system, a content management system (Orange) and the package SCORM to download the selected files of the different modules to the local system. User’s friendliness is ensured by the interface and the fact that each module will be supported by illustrations and interactive animations in order to visualise the module’s content for the user. The mathematical calculations in the modules are done with the Xmath Calculator.

The users of the system are not limited to educational world. Private companies, educational institutions and public offices connected to the education sector could use the different modules in a different way. However the system will also be open to users such as publishers, research scientists, engineers, etc. The database will be an “evolutionary system” that will be developed by the users, i.e. the users will be able to modify the existing modules and also to create entirely new modules. DMath information can be found at: <http://dmath.hibu.no/main.htm>.

The different mathematical modules drawn up cover a large part of the mathematical topics in Engineering Schools: Linear Algebra, Calculus in one and several variables, Ordinary Differential Equations, Numerical Methods, Statistics, Modelling, etc.

SciWriter, scientific editor designed for Soft4science, easily allows generating, using menu bars, scientific documents in different formats, especially the ones with the extension .xml [8]. Figure 1 shows part of one of the documents created using SciWriter, with the upper menu bars revealing the wide range of options provided by its use.

The database contains mathematical independent modules. It is necessary to do a clear distinction in the modules’ author way of working and of the potential users. From the author’s point of view, we have divided the content in small units, atoms, to be assembled afterwards in different ways. Different paragraphs: theory, examples, exercises, problems, applications, tests, exercises with web-Mathematica, etc. had been considered.

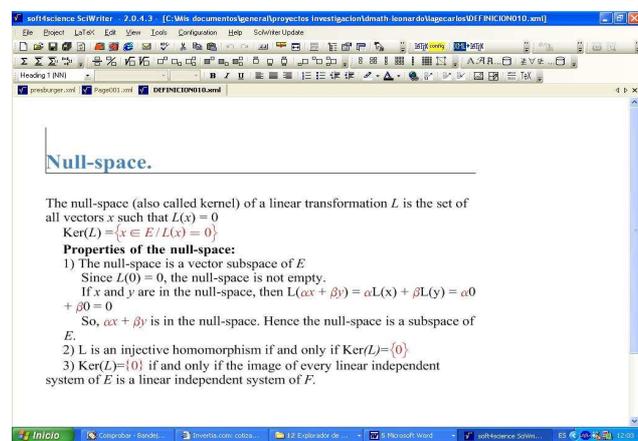


FIGURE 1. SCIWRITER DOCUMENT

By assembling atoms we form the molecules that can be used in different modules or units to be offered to the users.

By assembling atoms and molecules in different ways, the users can create their own documents according to the needs and the goals to be reached. Consequently, the versatility is very big and allows adapting to different needs, being able to develop courses with different levels of depth and generating documents to refresh contents, or doing self evaluation, showing applications, etc. The database is organized so that the system is simple to handle. The database structure is shown in figure 2. A colours code indicates in every moment the situation of different documents of the unit.

- The padlock symbol indicates that the atom or molecule is in the database and that it can be exported by SCORM, but can not be modified.

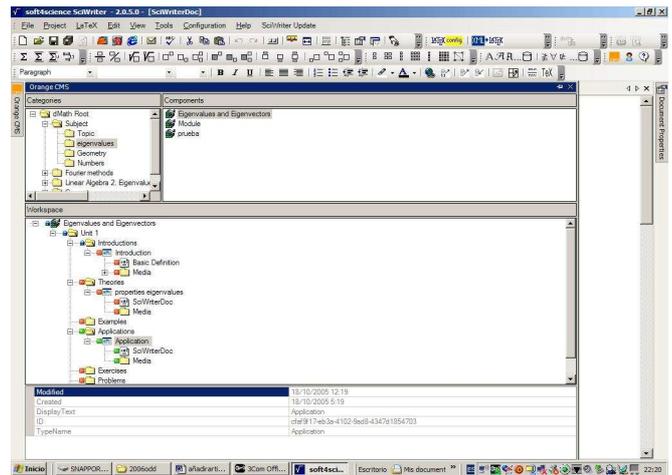


FIGURE 2. DATABASE

- The green colour indicates that the atom can be published.
- The red colour indicates that the atom is in use and can not be used by other users.
- The yellow colour indicates that the file is actually being created by the author and it has not been placed in the database.

Users can select what they like from every unit. The command Export has to be used for it and after that select the atoms/molecules that must appear in the designed unit. Hereby it is possible to adapt and use the documents according to the needs to be covered.

The students have used several modules according to their needs: For example in eigenvalues and eigenvector modules three different possibilities are offered:

A **theoretical module**, including definitions, propositions, etc. Also selected examples have been chosen. This module has been useful for students in a Linear Algebra Course for enhancing the theoretical aspects of spectral theory.

A **review module**: In this module only the main theoretical results have been included. The main goal has been to refresh concepts in Spectral theory.

An **application module**: Several applications have been shown, for example: Leontief models, dynamical systems, Google search algorithm, etc.

All the material developed in the project has been done in English. Though the language should be an aspect without influence for the utilization of the system, the tests carried out among our students reveals the difficulty to use non Spanish material. An idiomatic barrier appears for our students when using this material and this aspect could be the main handicap of this project. It is possible that the following project helps to remedy this fault.

## 6. THE EVLM PROJECT

The development of an **European Virtual Laboratory of Mathematics** and a Mathematical Centre, which allows the different European users to come in and search for expertise help for different mathematical problems that could arise both in the academic area and in the professional life, are the main goals of the project European Virtual Laboratory of Mathematics (EVLM) [4], also financed by the program

Leonardo da Vinci for the period 2006-2008 with the number 2006-SK/06/B/F/PP-177436. This project is coordinated by the Slovak Technical University (Slovakia) and the following academic institutions are involved: Plodivski University (Bulgaria), Salamanca University (Spain), Tilosilos Ltd. (Finland), Miskolc University (Hungary), University of Limerick (Ireland), Slovak Society for Geometry and Graphics (Slovakia), Coventry University (England) and University of West Bohemia (Czech Republic).

In this case, besides the official page of the project being in English, each of the institutions participating in the project has designed its National web page. The Spanish page can be found in <http://evlmspain.usal.es/>

The basic idea of this project is the development of the EUROPEAN VIRTUAL LABORATORY OF MATHEMATICS that will include a net of National Mathematical Centres located in the associate institutions. The EVLM's aim is to promote a better understanding of mathematical topics to be used in other disciplines, with a big component of mathematical background.

Every National Centre will develop a site (in the corresponding language) that will offer a virtual database with information about mathematical resources of virtual learning, elaborated by the partners and other sources (as previous projects financed by the European Union). Besides, the National Centres will offer a consultation service to those who want to know the new results in mathematics and in mathematical teaching.

The EVLM's target groups are:

- Students in a general way (from Secondary School up to PHD degrees). The elaborated resources will be available to get a better understanding of mathematical topics.
- Teachers and instructors. In the EVLM they will find resources for virtual education, the basic handbooks of their use and additional material.
- Researchers and scientists not belonging to academic areas. The EVLM will adapt and refresh the advanced mathematical tools necessary for the development of their work.

The final aims of the project are the following ones:

- To develop a catalogue of learning available materials in Mathematics.
- To establish a centralized database that could offer information and links on the catalogued resources.
- To offer an expert service of consultancy (face to face and on line) about the use of these materials.
- To promote the spreading and use of these resources offering the material's translation to the principal languages of the European Union.
- To offer an opportunity to the educational institutions to share with other European institutions the educational materials developed specifically for their purposes (often using funds of European programs).

It is necessary to remark that EVLM has two fundamental differences respect the previous project DMATH:

- The produced materials will be available not only in English, but also they will be in other languages

such as Spanish, Slovak, etc., which allows a more comfortable use of the didactic materials for instructors and students.

- The attention to the users, mainly students, not only is realized on-line, but users can take advantage of the possibility of doing face to face tutorials in the National Centres of Mathematics.

The general idea consists of the adjustment of already existing systems of other countries, for example the centre SIGMA of Coventry's University (Centre for excellence SIGMA in mathematics and statistics support), that it is possible to visit in <http://www.mathcentre.ac.uk/>. It is possible to access to the quoted site from the web page of the project.

In Salamanca University, the Centres are going to run in Zamora and Salamanca's campuses, beginning in the academic course 2007-08. The first one of them will be in the Polytechnic of Zamora and the second one in the Science Faculty of Salamanca. The participation of all departments that have a teaching responsibility in mathematics (in different levels and studies imparted in the university) will be promoted in these centres.

During the first year a survey concerning several aspects such as the use of services of this Centre and the satisfaction's level (instructors and students) will be analysed.

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