Nuggets of Mechanical Engineering – Revisit of the Free-Body Diagram Analysis and Force Flow Concept

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***Abstract* - Several key concepts in mechanical engineering, such as free-body diagram analysis, force flow concept, stiffness network, observing coordinates for kinematics, stress analysis, electric circuit analysis, tolerancing, etc, often present great difficulties to students. In this paper, the author starts an attempt to revisit some fundamentals. It will be called nuggets of mechanical engineering. Although no new theories have been developed, the presented thoughts and methods might be useful to help ME students to become more fundamentally sound. In this first of a series of planned papers, the author focuses on the free-body diagram, the force flow concept, and spring network model. The force flow concept is in particular powerful in handling over-constraint systems, such as those in most practical machinery. Several practical examples are used for illustration.**

Index Terms – Free-body diagram, Force flow concept, Stiffness, Spring Network.

INTRODUCTION

Mechanical Engineering students have the burden to study a wide range of subjects, such as sold mechanics, dynamic and control, fluid mechanics, thermodynamics, design, etc. The fundamental concepts covered in the first and second years of study are often most important and can affect their proficiency in handling more difficult subjects in their junior and senior years. Several key concepts, such as free-body diagram analysis, force flow concept, stiffness network, observing coordinates for kinematics, and stress analysis, often present great difficulties to students. Sometimes, incorrect perception of these important fundamental concepts does more harm than good.

In this paper, the author revisits some fundamentals. It will be called nuggets of mechanical engineering. Although no new theories are developed, the presented thoughts and methods might be useful to help ME students to become more fundamentally sound. In this first of a series of planned papers, the author will focus on the free-body diagram, the force flow concept, and the spring network. The force flow concept is particularly powerful in handling over-constraint cases, as in most practical machinery. Several practical examples are used to further illustrate the usefulness of these techniques.

Specific nuggets that would be presented are listed as follows:

- A simple procedure for counting all forces in the free-body diagram analysis.
- Filling-in instead of cutting-off in the free-body diagram analysis.
- Procedures to construct a force flow chain.
- Force flow chain case #1: without external forces.
- Force flow chain case #2: with external forces.
- Procedures to convert force flow diagrams into a spring network.
- Sense of comparative stiffness.
- Spring network analysis considering initial states of the springs.
- Practical examples.

REVISIT OF FREE-BODY DIAGRAM ANALYSIS

I. The ABCC Procedures

It is true that conducting the free-body diagram analysis is easy but it is also true that the analysis is often incomplete or wrong if it is not conducted carefully. A common mistake is that one or more forces are overlooked during the analysis. For minimize the chance of overlooking some forces, the following procedures can be suggested to students. It is denoted as the ABCC procedure. Each letter of these four letters, ABCC, represents a specific force type. By following the ABCC procedure, a student would go through every force type, thus minimizing the chance of overlooking any of them.

- **Letter "A"** stands for applied forces or external forces to the system. These are the forces applied to the system which can be static or as functions of time. Of course, the applied force can also be in the form of bending moment, torque or distributed forces.
- **Letter "B"** stands for body forces, which include the forces due to gravitational, electric, magnetic fields, etc. In most cases, body forces can be neglected to simplify the analysis but it is of value to go through the thought

process of identifying them and then taking them off if they can be ignored.

- **The first Letter "C"** stands for contact forces. When we conduct the free-body diagram analysis, we often need to remove some constraints. If the constraint is not a permanent type, the forces required to replace these constraints will be called the contact forces. For example, a ball resting on a flat surface is not permanently bond to the surface but merely in contact with the surface. The contact forces to be considered are the normal contact force, which is normal to the contact surface, and the tangential contact force, which is tangential to the contact surface. Normally, no moments need to be considered when a contact-type constraint is removed.
- **The second Letter "C"** stands for constraint forces. Here, the constraint forces are defined to the forces and/or moments needed to replace a constraint where two bodies are bonded permanently. These constraints include pin supports, clamped supports, welded joints, or splitting a component into two parts.

The example below, from a Statics textbook [1], is used to illustrate the effectiveness of the free-body diagram analysis following the ABCC procedure. When an ME student is asked to produce a correct free-body diagram (for example, the one shown in the left-bottom diagram in Figure (1)) based on a given problem (for example, the one shown in the left-upper diagram of Figure (1)), he or she usually tries to label all related forces without following any specific sequences. Quite often, one or two forces are neglected during the analysis, leading to incorrect answers. By the following the suggested ABCC procedure, students will go through all four different types of forces in an organized way. Therefore, they can reduce the chance of missing any forces. The diagrams on the right-hand side of Figure (1) break down the four A-B-C-C steps in order to derive the correct free-body diagram. For the problem show in Figure (1), it might appear trivial in applying the ABCC procedure; however, following the suggested procedure often makes students more disciplined and organized in their analysis.

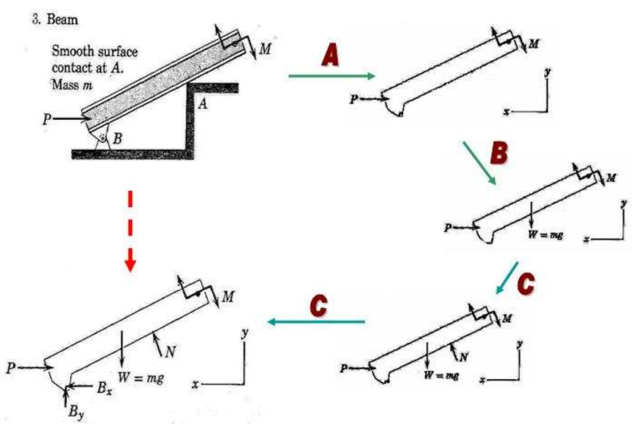

Figure (1)
Free-body diagram analysis following the ABCC procedures.

II. Filling-in instead of Cutting-off

Almost all the examples in standard textbooks regarding the free-body diagram analysis are related to removing constraints or cutting off a part of a system and replacing them with equivalent forces and/or moments. However, there are cases where we can fill in a system to make it geometrically simpler for analysis. The procedures involved for this filling-in analysis in general follow the reverse order of the conventional steps, in which a part is removed and forces are introduced. We will use a bearing strain field analysis example to illustrate the filling-in procedures for the reversed free-body diagram analysis. This filling-in procedure was suggested by Prof. J. Barber [2] in dealing with a spindle bearing problem in [3].

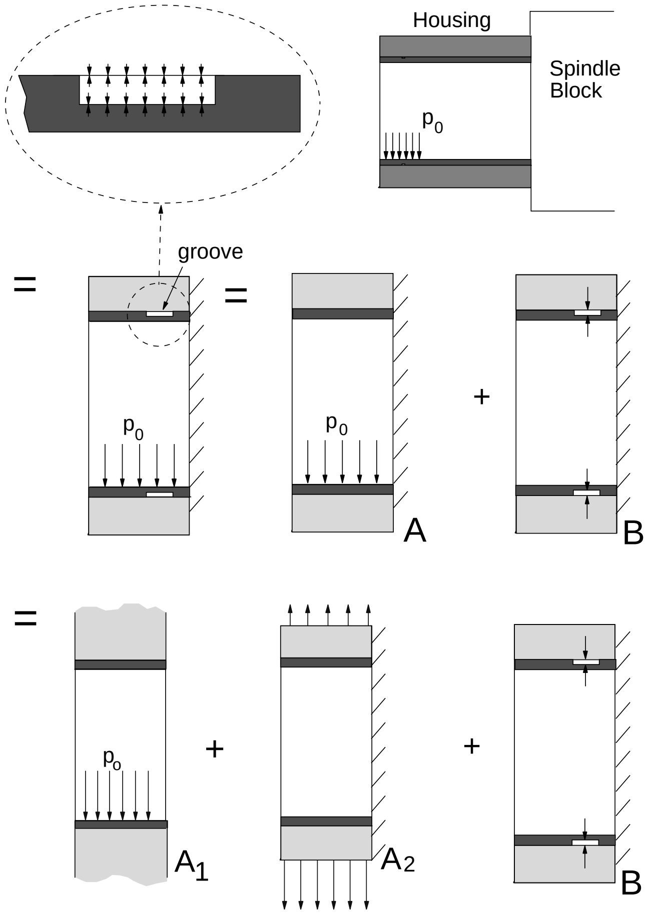

FIGURE 2
Bearing sensor design with the filling-in procedure for the reversed free-body diagram analysis.

As shown in Figure (2), a set of strain gages are to be mounted on the outside surface of the bearing outer ring in a machine tool spindle. In order to mount these sensors, it is necessary to grind a groove around the outer ring to accommodate strain gages. It is then desirable to estimate the strain field at the sensor location for the sensor design. Figure (2) illustrates the steps of the free-body diagram analysis with the filling-in technique. First, the spindle bearing housing is isolated as a cantilever beam with an external loading, P_o. Notice that the groove around the outer ring is identified in Figure (2). As shown in the enlarged view of the groove, we now introduce force pairs on the walls of the groove. With superposition, we split the original case into cases A and B. Case A contains P_o and the forces inside the groove which are pointing outwards, while case B contains only those forces outside of the groove pointing inwards. If we choose these force pairs in correct values, those forces inside the groove pointing outward can be considered as the reaction forces if there is no groove; in other words, we can fill-in the groove to replace those forces inside the groove. This is exactly reverse of regular free-body analysis in which body is removed and replaced by the equivalent forces and/or moments.

We can take the same step further by introducing force pairs around the outer surface of the housing in case A. Following the same procedures, the forces pointing inwards are now replaced as an infinite body as case A_1 and those forces pointing outwards forms case A_2. As discussed in [3], the strain field at the sensor location inside the groove can be determined with acceptable accuracy by considering case A_1 only. The advantage of all these filling-in steps is now clear as case A_1 has a classical, closed-form solution [4]. Therefore, without jumping to the FEM codes, we can have a credible estimate of the strain field of the entire system. If high accuracy is desired, one can always turn to the FEM codes but the result of the above analysis can still be used for cross-checking to debug errors of the FEM programming.

FORCE FLOW CONCEPT AND SPRING NETWORK MODEL

I. Force Flow Concept and Force Flow Loop

Although the free-body diagram analysis is useful and essential in the force analysis of mechanical systems, it can become highly complicated if a system contains many components and/or the system is over constrained. In practice, most machinery contains over-constraint configurations. To deal with these more complicated cases, the force flow concept, to be introduced in this section, is particularly powerful. An excellent description of the force flow concept can be found in [5]. It showed that the force flow concept can be used to effectively locate critical sections of a machine structure, to analyze redundant ductile structures, and to determine stress distribution within axially loaded members [5]. In this paper, we will only provide a brief introduction to the force flow concept so that we can avoid repeating the materials that have been properly covered in [5]. We will, instead, focus on the conversion of a force flow loop into a spring network which makes the analysis of redundant ductile structures more intuitive.

To apply the force flow concept, we employ an orderly procedure of following the "lines of force" through the various components of a mechanical system [5]. The two presses in Figure (3) from [5] provide an excellent example for demonstrating the effectiveness of the force flow concept. The red lines in Figure (3) represent the path through which the force flows, and these paths eventually connect and form a loop. By looking the force flow loop, it becomes very easy to identify if a component is under loading without the need to conduct the free-body diagram analysis. In addition to the procedures described in [5], we propose to add a letter, "C", for identifying components under compression and a letter, "T", for those under tension. The size of the force flow loop is also an indication of the effectiveness of a machine design. As marked in Figure (3), the press on the left employs a larger force-flow loop. A larger force flow loop usually indicates more components are under loading. For the press design on the left in Figure (3), the force flow passes through all sections of the frame which means that these sections are carrying loads; as a result, they require heavy construction which increases the dead weight of the overall press significantly. An even greater deficiency of this design is that their long power screws are loaded in compression, which invokes immediate concern of buckling, which drastically reduced the press load capacity.

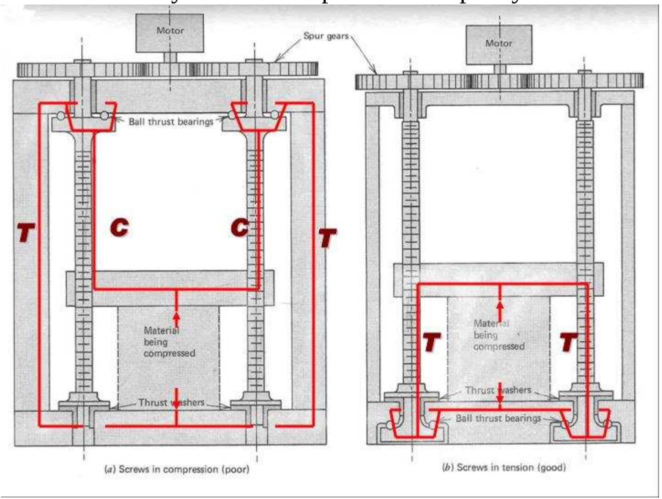

Figure (3)
Application of force-flow concept for design [5].

In comparison, the design of the press on the right involves of a smaller loop. All sections of the frame are not load carrying. In addition, its long power screws are loaded in tension instead of compression. As a result, the press on the right has a light deadweight, is of lower material cost, and has a higher load capacity because there is no concern of buckling.

Such quick and insightful observation of two press designs is made possible with the use of the force-flow concept. If students can only analyze this problem through the conventional free-body diagram analysis, it would be a highly complicated task. Even if students are able to disassemble the entire press into several free-body diagrams, the problem will still be unsolvable because there will be too many unknown forces involved in those free-body diagrams.

II. Spring Network converted from Force-Flow Loop

The force flow concept not only can provide a quick and insightful design evaluation, it can also be used to determine forces acting on different components. As mentioned above, using the conventional method, students will be facing too many unknown forces to continue their analysis of the problem.

In order to determine the unknown forces in over-constraint problems, it is necessary to convert a force-flow loop into a spring network model. Spring network models have been used to analyze loading capacity of joints, in particular bolted joints in [6]. However, no intuitive procedures are provided in [6] for constructing a correct spring network. In this paper, we will show that following a force flow loop, a spring network model can be readily constructed. For illustration purposes, we will use another example from [5]. Note that, in [5], this example was analyzed using the conventional free-body diagram analysis, and students often have difficulties to comprehend the analysis because it involves an over-constraint problem. As shown in Figure (4), two sealed tanks with high pressure gas inside are illustrated. The tank on the left has a soft gasket to seal between the cover and the tank body, while the one on the right uses an O-ring with the cover directly in contact with the tank body. For the tank on the right, it is important that the mating surfaces of the cover and the tank body are machined within a proper flatness tolerance to maintain proper seal.

As illustrated in Figure (4), a force flow loop is plotted for each tank. In addition, we draw a slender rectangular on each force flow path to represent the component that the force passes through. With this extension, the new force flow loop can be readily converted into a spring network model. This is done simply by replacing each component (the slender rectangular) with an equivalent spring. One thing to remember is that an equivalent spring is also needed for the interface between two mating surfaces (see below). As shown in Figure (4), there are many equivalent springs to consider. Because the resulting spring model can be highly complicated when many components are involved, it will be beneficial to conduct some preliminary rigidity analysis to simplify the spring network. Therefore, instead of drawing a complicated spring network by including every equivalent spring, we will first identify the relative spring constant of each component. Those with very high relative spring constant can be treated as rigid and be excluded from the analysis.

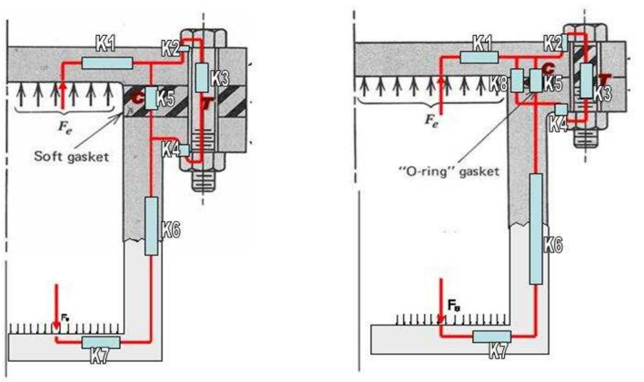

FIGURE 4
Force-flow loop and its conversion to spring network model.

As shown in Figure (4), there are seven springs, K1 to K7, to be considered. For springs K1 and K7, they are related to the bending mode of doubly clamped thick and short structures. We can safely assume that they are of very high rigidity and exclude them from the analysis. Springs K2 and K4 are due to the Hertzian contact stress between two flat surfaces. The equivalent spring constant between two flat metal surfaces is usually very high [7] when it is compared with that of spring K3 which is related to the tension of a slender bolt. Because K3 is connected in series with K2 and K4, the combined stiffness of springs is dominated by the softer spring, thus, K3. As a result, we can exclude K2 and K4 from analysis. For the tank on the right, spring K8 is related to the contact stress between the cover and the tank body, while spring K5 is related the O-ring. As these two springs are in parallel, K8 will be dominating; thus K5 can be excluded from analysis for the tank on the right. On the other hand, for the tank on the left, spring K5, which represents the soft gasket, must be considered because it does not connected in series or in parallel with any other springs. Finally, spring K6 is related to the tank body, which is of very high stiffness, and, thus, can be considered as rigid.

After the above analysis of relative stiffness, the force flow loops in Figure (4) can be converted into much simpler spring network models as shown in Figure (5).

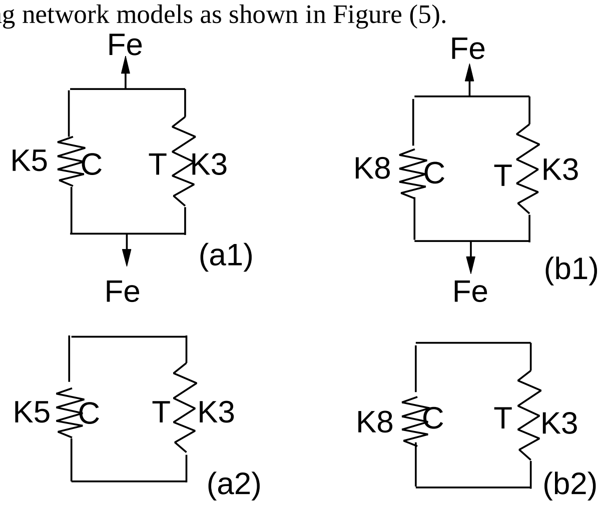

FIGURE 5
Simplified spring network models for the tank problems

Now let's consider the tank on the left as represented by the spring network model of Figure (5a1). Before analyzing Figure (5a1), we would consider the case before F_e is applied. This is the case when the tank cover is mounted and the bolt is tightened to a pre-determined load. This preloading practice causes the bolts to be extended while the soft gasket is compressed. After preloading, the tank is represented as the spring network model in Figure (5a2). Let the initial deflection of the bolt be δ_{30} and the initial compression of the gasket be -δ_{50}, the preloading force, F_{po}, can be expressed as

$$F_{po} = K_3\delta_{30} = K_5\delta_{50}. \tag{1}$$

After filling in high pressure gas into the tank, the external force F_e is now applied and is represented by the spring network model of Figure (5a1). The external force, F_e, causes the bolt to be further extended by a deflection δ, while causing the soft gasket to be less compressed by the same amount, δ. In other words, F_e is taken up by both K_3 and K_5,

$$F_e = \Delta F_3 + \Delta F_5 = \delta K_3 + \delta K_5 \quad (2)$$

and the forces acting on K_3 and K_5 now become

$$F_3 = (\delta_{30} + \delta)\ K_3 = F_{po} + \Delta F_3$$
$$F_5 = (-\delta_{50} + \delta)\ K_5 = -F_{po} + \Delta F_5 \quad (3)$$

From Equations (2) and (3), it follows that

$$\Delta F_3 = F_e K_3 /(K_3 + K_5)$$
$$\Delta F_5 = F_e K_5 /(K_3 + K_5) \quad (4)$$

As a soft gasket is used, it is true that $K_3 >> K_5$; therefore,

$$\Delta F_3 \cong F_e$$
$$\Delta F_5 \cong 0 \quad (5)$$

Equation (5) indicates that most of the external loading is taken up by the bolt by further extending it. Similar analysis can be conducted for the tank on the right (Figures (5b1) and (5b2)). Because $K_8 >> K_3$, we can reach a completely different result as

$$\Delta F_8 \cong F_e$$
$$\Delta F_3 \cong 0 \quad (6)$$

Equation (6) indicates that the external loading is now mainly used to release the compression between the mating surfaces of the tank cover and the tank body. Finally, if a hard gasket is used, K_3 and K_5 can be in the similar order of magnitude. As a result, a portion of the external force is used to release the compression force and the rest is used to increase the tension of the bolt as indicated by Equation (4).

When the author presented this problem to students, almost all students intuitively assumed that the bolt is taking up the entire external force without realizing that it also takes effort to release compression. By constructing a force flow loop and then converting it to a spring network model, the problem becomes straightforward and easy to understand.

APPLICATION TO MACHINE TOOL SPINDLE DESIGN AND ANALYSIS

Let's apply the force flow concept and the above example to a practical machine tool spindle design problem. Figure (6) is a precision tool-room surface grinder spindle from [8].

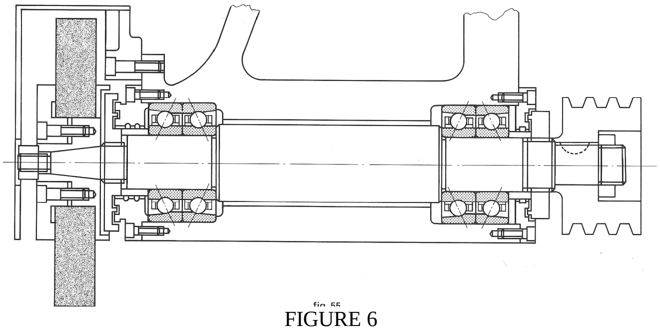

FIGURE 6

Precision Tool-room Surface Grinder Spindle

As this spindle is designed for cylindrical grinding, the grinding forces are mainly in the radial direction and the axial direction if the front face of the grinder is also used for grinding. The radial grinding force and its resulting bending moment to the spindle are taken up by both the front and the rear bearings. The reaction forces in the radial direction of the bearings can be easily determined by assuming the bearings are simple pin-supports. However, for the axial force, it is more complicated. First, we recognize that on the front bearing set can counteract the axial force because the rear bearing set is designed to be floating axially so that it can accommodate thermal expansions. Secondly, the bearing sets in this precision spindle are preloaded. The preloading and the thermally induced preload issues have been investigated in a series of papers [9-15].

The loading condition of the front bearing set under the preloading force and the thrust force has been discussed in [14] and we will find it very similar to the example discussed in Figures (4) and (5).

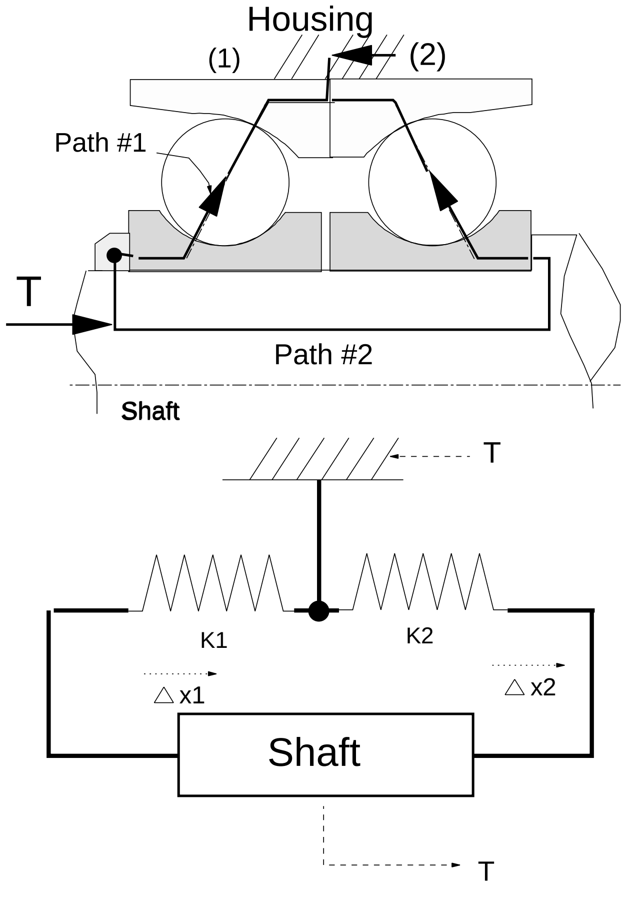

FIGURE 7

Thrust load analysis for back-to-back Bearing set.

As shown in Figure (7), during the preloading stage (i.e., the thrust load T is zero), a compressive preload F_{po} is created and both spring K1 and K2 (similar to Figure (5a2)). When the thrust force, T, is applied, it is used to increase the compression in K1 (loading), while to release the compression in K2 (unloading). This is similar to the tank example (Figure (5a1)). The spring network in Figure (7) provides a visual analysis for these loading conditions. As the stiffness of the bearing is mainly due to the elastic deformation of the ball, experimental results from [8] show that, for ball bearings, the average loading stiffness of K1 is approximately twice the unloading stiffness for K2. Therefore,

$$K_1 = 2\ K_2 \qquad (7)$$

Following Equation (4), we have

$$\Delta F_1 \cong 2/3\ T$$
$$\Delta F_5 \cong 1/3\ \mathrm{T} \qquad (8)$$

The result of Equation (8) indicates that 2/3 of the thrust load is used to further increase the force on the front bearing K1, while one third of it is used for unloading the rear bearing K2. As a result, the front bearing K1 is more susceptible to failures. To release overloading of the front bearing, different bearing configuration such as those shown in Figure (8) can be employed [8]. From the above analysis, it becomes clear why the design in Figure (8) is effective.

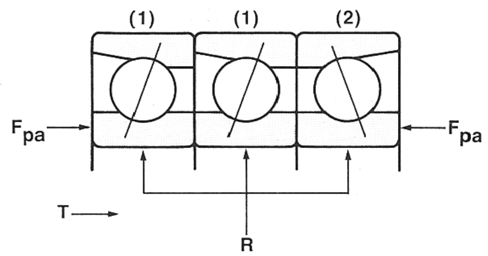

FIGURE 8
Thrust load analysis for triple unit Bearing set.

For the triple unit configuration of Figure (8), it becomes clear that 80% of the thrust load is used for loading the first two bearing (40% each), while 20% is used for unloading the rear bearing [8]. A modern medium speed spindle for turning machines or machining centers is shown in Figure (9) in this configuration [8].

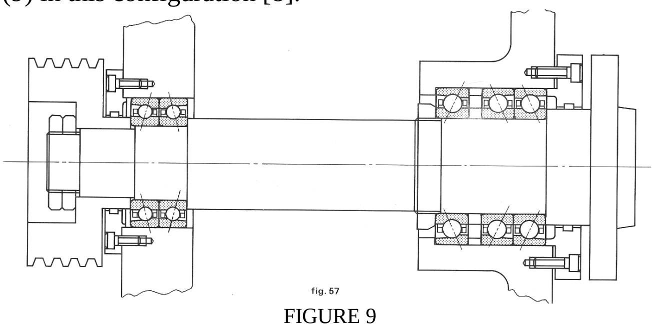

FIGURE 9
A typical spindle of modern turning machines or machining centers with a triple-unit bearing arrangement.

CONCLUSION

In this first of a series of papers the author plans to write, the author revisit the free-body diagram, the force flow concept, and spring network model, and their applications to handle over-constraint problems in practical machinery. Although no new theories have been developed, the presented thoughts and methods could be useful to help ME students to become more fundamentally sound.

ACKNOWLEDGMENT

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