REVISITING MATRIX THEORY AND ELECTRIC CIRCUIT ANALYSIS

Rogelio Palomera-Garcia
U. of Puerto Rico-Mayaguez,
Dept. of Electrical and Comp. Eng.
P. O. Box 9042, Mayaguez PR 00682
palomera@ece.uprm.edu

Abstract - **Matrix Theory has long been applied to many branches of engineering. However, numerical difficulties have limited access to students, especially at the elementary level. This limitation has partially been removed with the development of canned software and modern scientific calculators which operate matrices. However, the approach with matrices ignores this and still follows traditional ways. This work presents an alternative view of matrix applications to electric circuit analysis, linking matrices and equations with circuit solutions, with heavy emphasis on appropriate setting of equations and interpretation of results. As a result, a more effective use is made to obtain applications in areas that usually require more work or symbolic analysis, such as when dealing with symbolic sources, equivalent port representations, two ports, etc.**

Index Terms – Matrix analysis, Circuit Theory, Systems education

INTRODUCTION

Matrices have been used in circuit analysis for a long time [1-3]. Yet, matrices have been more useful for "advanced" courses or for computed aided oriented books [4, 5]. Popular introductory circuit books usually make limited use of them when coming to the mathematical analysis of the circuits. The main emphasis is either for the solution of a set of equations using Cramer's or determinants [6, 7] or mainly for preparing the equations for use in Matlab® [8]. Gottling [9] focuses more on the use of matrices, but emphasis is definitely on Matlab programming.

Almost all teachers would agree that the use of matrices in the introductory courses, without more than a precalculus level treatment, would focus in the solution of the set of linear equations, and this is in fact how matrices are used at this level and will certainly be most of the time. Properties of matrices are also linked with some properties of circuits, although this relationship is seldom considered.

The main problem with matrices in introductory courses is of numerical nature since hand and pencil analysis becomes usually more complicated, except for simple cases; Cramer's rule is impractical for more than three equations. Software tools such as Matlab®, Mathematica® or MathCad® alleviate, but do not necessarily solve the problem, since there are still practical problems associated for home study such as the program availability, the computer, and so on.

On the other side, modern graphical calculators such as the TI-89™, HP49™ and many others have matrix operations included, so students count now with a computation capacity allowing them to focus on the solution of the equations as easily as with the use of expensive software and laptops.

However, the power of such tools can only be appreciated at elementary courses if we reconsider the way matrices are used in circuit analysis. True, the main use of matrices at this level will still be the solution of equations. Yet, the equations have more power than usually thought when properly considered and the solutions adequately interpreted. Most texts use matrices as an intermediate tool in a solution process which becomes invisible at the moment the circuit includes elements such as symbolic sources or multiple sources as functions of time in resistive circuits, etc. In these situations, the student without access to calculators or programs with symbolic manipulation power becomes lost for circuits with a little more complexity.

The purpose of this work is to look at elementary matrix theory, or better, at the use of matrices to solve equations, under a renewed perspective, which puts at the reach of early classes in circuits the tools of matrices in a formal and useful way. For all practical purposes, the approach given here is extendable to any linear system, mechanical, electrical, etc.

No new theorem or theory is introduced, only an approach from a new perspective. Among features of this approach attractive for using matrices early we can mention:

- There is no need to consider complicated notations such as sub matrices and operations with them. [3,4,10]
- Only numerical operations are involved, so there is no need for programs or calculators with symbolic capabilities.
- There is a direct and immediate connection between theoretical concepts in linear circuits and how equations are set up and solutions interpreted.

The theory and examples presented in this paper are limited to resistive circuits due to space limitations. The method and numerical procedures are of course extendable to the phasor domain. Other topics that stem from an extension of this approach are left outside this paper. The setting of equations directly using characteristics of equations is not considered, since the rules are already popular and can be looked elsewhere, like in [7, p. 90, p. 105]

This approach does not pretend to substitute, but to complement, other methods, including hand and pencil procedures, too useful in the development of intuition, or more convenient under different circumstances.

MATHEMATICAL BACKGROUND

A set of linear mathematical equations is written as

$$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2$$
$$\ldots$$
$$a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n = b_n \qquad (1)$$

In matrix form, this can be written as

$$\mathbf{Ax=b} \qquad (2)$$

where

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & .. & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \cdots \\ x_n \end{bmatrix} \text{ and } b = \begin{bmatrix} b_1 \\ b_2 \\ \cdots \\ b_n \end{bmatrix}$$

Here, the coefficient matrix $\mathbf{A}$ and the vector $\mathbf{b}$ are assumed to be known, with $\mathbf{A}$ being invertible. The solution for $\mathbf{x}$ in (2) is then found with

$$\mathbf{x = A^{-1}b} \qquad (3)$$

This result can be found in any standard text using matrices to solve the set (1). Now, our new focus goes into a common form for the set of equations in circuit applications, namely, when the right side elements of (1) are in fact linear combinations themselves. That is, the form

$$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_{11}z_1 + b_{12}z_2 + \cdots + b_{1m}z_m$$
$$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_{21}z_1 + b_{22}z_2 + \cdots + b_{2m}z_m$$
$$\ldots$$
$$a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n = b_{n1}z_1 + b_{n2}z_2 + \cdots + b_{nm}z_m$$

$$\qquad (4)$$

.

Although the main difference between (4) and (1) is that now the right hand side of each equation is a linear combination of the $z_i's$, and thus no particular interest seems to arise, very often the $z_i's$ are parameters of interest, such as other unknowns, special functions, constants, etc. In this case, it is useful to discuss the equations under other perspective. Using column vectors $\mathbf{b}_i = \begin{bmatrix} b_{1i} & b_{2i} & \cdots & b_{ni} \end{bmatrix}^T \quad i = 1, 2 \ldots, m$, (4) can now be expressed as

$$\mathbf{Ax} = \mathbf{b}_1 z_1 + \cdots \mathbf{b}_m z_m \qquad (5)$$

or else in the form

$$\mathbf{Ax=Bz} \qquad (6)$$

where $\mathbf{B}$ is the matrix with the $\mathbf{b}$ vectors as columns, and $\mathbf{z}$ the vector of the parameters z_i.

With the notation of (5) or (6), the solution for the set of equations becomes

$$\mathbf{x} = \mathbf{A}^{-1}\mathbf{b}_1 z_1 + \mathbf{A}^{-1}\mathbf{b}_2 z_2 + \cdots + \mathbf{A}^{-1}\mathbf{b}_m z_m = \mathbf{A}^{-1}\mathbf{Bz} \qquad (7)$$

The form of system (4) is very common in linear circuits; and for that matter, in any linear system. Some examples will illustrate the use of it later. For the moment, an important consequence of (7) is the fact that the solution for any variable x_i in (4) is of the form

$$x_i = k_{i1}z_1 + k_{i2}z_2 + \cdots + k_{im}z_m \qquad (8)$$

where k_{ih} is the i-th element of the vector $\mathbf{A}^{-1}\mathbf{b}_h$ or the element in the i-th row and h-th column of $\mathbf{A}^{-1}\mathbf{B}$.

Notice that from the algorithmic point of view, the solution of system (4) is reduced to numerical matrix multiplication and and appropriate interpretation. A simple example will illustrate this remark

Example 1: Solve the following system for variables x and y:

$$5x + 12y = 8\cos(3t) + 7e^{-2t}$$
$$3x + 7y = -2\cos(3t) + 5e^{-2t}$$

Identifying $z_1 = \cos(3t)$ and $z_2 = e^{-2t}$, the solution can be found, according to (7), with the operation

$$\begin{bmatrix} 5 & 12 \\ 3 & 7 \end{bmatrix}^{-1} \begin{bmatrix} 8 & 7 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} -80 & 11 \\ 34 & -4 \end{bmatrix}$$

This numerical result can now be interpreted as $x = -80\cos(3t) + 11e^{-2t}$ and $y = 34\cos(3t) - 4e^{-2t}$. **End**

The example, although simple enough to be worked by hand, illustrates the procedure for any order and pretends to bring to the attention two points: first, by associating the right hand side to a matrix and not to a vector, we reduce the operations to a numerical procedure not involving the parameter t or the cosine or exponential functions; second, the final result as well as the method statement requires interpretation of computer (calculator) results; this is aligned with the skills ABET lists for engineering programs[11].

The coefficients k_{ih} in (8) can be interpreted in a suitable way for applications and for use in even lower tech calculators, which may not necessarily have matrix operations but just a utility for solving linear equations of form (1):

$$\begin{bmatrix} \text{The coefficient } k_{ih} \text{ in formula (8) is the solution} \\ \text{for } x_i \text{ in system (4) for the special case when } z_h = 1 \\ \text{and } z_k = 0 \text{ for } k \neq h \end{bmatrix} \quad (9)$$

A first consequence of (9) is that system (4) can be solved using systems of the form (1) m times, each one with the elements of the appropriate vector $\mathbf{b}_k$. Second, (9) naturally leads us to the basic properties of linearity, that is, the homogeneity and additivity properties.

INTERPRETING THE MATHEMATICAL RESULTS FOR APPLICATIONS

The form of system (4) is the general form of equations with respect to voltages and currents. This follows from the very fact that (a) Kirchhoff's equations are linear in nature – and hence, linear also for non linear networks, - and (b) that the element equations are linear by definition. The sources may be of any kind, symbolic or dependent of time, as far as they do not introduce non linear relationships for voltages and/or currents. This feature, together with interpretation (9), will allow us to work with symbolic sources, time-dependent sources, superposition, network functions and other characteristics, and many more situations. It will be easier to illustrate these remarks using examples.

I. Symbolic Sources, Superposition and functions of time.

Example 2 (symbolic sources). For the circuit of figure 1, solve for the node potentials in terms the sources E1 and I2.

We will use the common concept of supernode [7, 8] for nodes 1 and 3, taking equations for nodes 4 and the "supernode" (1,3) and add the dependent source equation. The reader may check the following equations, where zero coefficient terms were omitted for simplicity:

Supernode (1,3):

$$\left(\frac{1}{4}+\frac{1}{20}\right)V_1 + \left(\frac{1}{40}+\frac{1}{2}+\frac{1}{8}\right)V_3 - \frac{1}{8}V_4 = \left(\frac{1}{4}+\frac{1}{2}\right)E_1 - I_2 \quad (10)$$

Node 4: $\qquad -\frac{1}{8}V_3 + \left(\frac{1}{8}+\frac{1}{80}\right)V_4 = I_2 \qquad (11)$

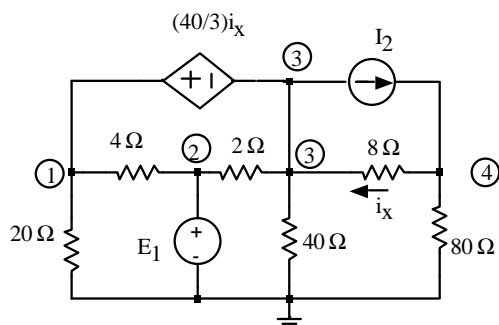Dependent source: $V_1 + \left(\frac{5}{3}-1\right)V_3 - \frac{5}{3}V_4 = 0 \qquad (12)$

FIGURE 1 CIRCUIT FOR EXAMPLE 2

We see that equations (10)-(12) are of the form (4). We can therefore write the matrix form (5), **Ax=Bz**

$$\begin{bmatrix} \frac{1}{4}+\frac{1}{20} & \frac{1}{40}+\frac{1}{2}+\frac{1}{8} & -\frac{1}{8} \\ 0 & -\frac{1}{8} & \frac{1}{8}+\frac{1}{80} \\ 1 & \frac{5}{3}-1 & -\frac{5}{3} \end{bmatrix} \begin{bmatrix} V_1 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} \frac{1}{4}+\frac{1}{2} & -1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} E_1 \\ I_2 \end{bmatrix}$$

In this expression the operations in the elements have not been realized to illustrate the fact that, when using calculators, the operations do not need to be carried out previously. We can now work the operation

$$\begin{bmatrix} \frac{1}{4}+\frac{1}{20} & \frac{1}{40}+\frac{1}{2}+\frac{1}{8} & -\frac{1}{8} \\ 0 & -\frac{1}{8} & \frac{1}{8}+\frac{1}{80} \\ 1 & \frac{5}{3}-1 & -\frac{5}{3} \end{bmatrix}^{-1} \begin{bmatrix} \frac{1}{4}+\frac{1}{2} & -1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} =$$

$$\begin{bmatrix} 0.805 & 8.123 \\ 0.948 & -4.713 \\ 0.862 & 2.989 \end{bmatrix}$$

The <u>numerical</u> result is next interpreted as

$V_1 = 0.805E_1 + 8.123\ I_2,\quad V_3 = 0.948E_1 - 4.713\ I_2$ and
$V_4 = 0.862E_1 + 2.989\ I_2$

End of Example

Example 2 can be similarly worked for time dependent sources the type $E_1 = kf(t)$ and $E_2 = mg(t)$. In this case, the constants k and g are introduced as factors in the respective columns in **B** and the time functions then added when results are interpreted, as it was illustrated in Example 1.

Example 2 also illustrates the superposition principle from the very set up of the equations. In fact, one can see that the first column of the matrix **B** of example 2 is the situation in which $I2=0$. This can be double checked by open-circuiting the source and writing again the equations. Similarly, the second column corresponds to the case $E1=0$, which can be verified short-circuiting node 2 to ground and writing the equations. Consequently, the first and second columns of the resulting matrix show the individual contributions of each source. Thus, when individual contributions need to be highlighted, write a separate column for the sources instead of adding all of them in one term.

Interestingly enough, this also illustrates the fact that the matrix coefficient may written after turning off all the independent source. This has important consequences in transformations and applications of circuits.

II. Network Functions

Equation (8) and interpretation (9) can be directly related to transfer and port functions. If the z_i's in (4) are voltage or current input magnitudes, and x_h is a response voltage or current, then, according to (9), k_{hi} can be interpreted as the

network function (either transfer or port, depending on the variables involved)

$$k_{hi} = \frac{x_h}{z_i}\bigg|_{z_m=0,\ m\neq i} \tag{13}$$

Notice that, mathematically, not all variables in (4) need to be responses. The network function for only one input is worked similarly, making the input value 1 and interpreting the results accordingly. Again, let us use an example providing an ample illustration.

Example 3. For the sub circuit of Fig. 2, find the input resistance Vin/Iin, as well as the transfer functions using nodal equations.

$$\frac{V_a}{I_{in}}, \frac{V_b}{I_{in}}, \frac{I_a}{I_{in}}, \frac{I_b}{I_{in}}, \frac{V_a}{V_{in}}, \frac{V_b}{V_{in}}, \frac{I_a}{V_{in}}, \frac{I_b}{V_{in}}$$

To set up the equations, we prepare the circuit introducing an "input" Iin=1A, assign nodes and ground as shown in Fig. 3. With this setting, Vin=V_1, Va=V_2 and Vb=V_2-V_3 , Ia=V2/2000 and Ib=Vb/5000.
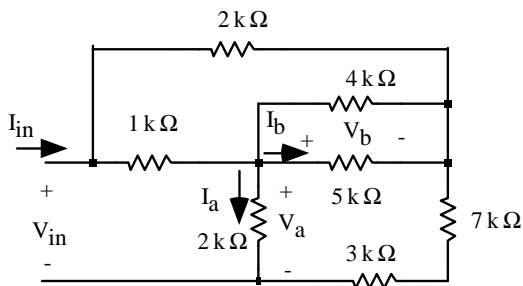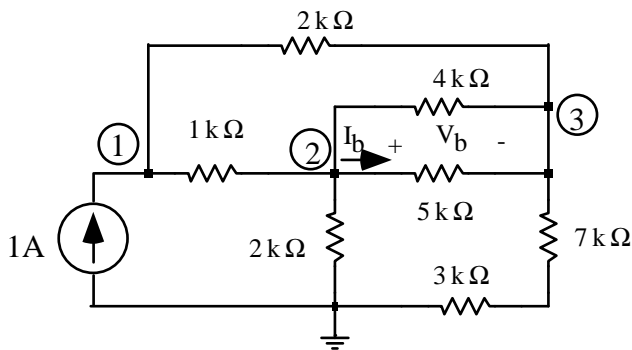

FIGURE 2   CIRCUIT FOR EXAMPLE 3

FIGURE 3   CIRCUIT PREPARED FOR ANALYSIS

To solve for the required functions, from (9) and (13) we conclude that these values will be numerically equal to the functions Rin, Va/Iin, Vb/Iin, Ia/Iin and Ib/Iin. Dividing all the transfer functions by Vin will yield then the rest of the functions. Setting up the equations we get

$$\begin{bmatrix} \frac{1}{1k}+\frac{1}{2k} & -\frac{1}{1} & -\frac{1}{2k} \\ -\frac{1}{1k} & \frac{1}{1k}+\frac{1}{2k}+\frac{1}{4k}+\frac{1}{5k} & -\frac{1}{4k}-\frac{1}{5k} \\ -\frac{1}{2k} & -\frac{1}{4k}-\frac{1}{5k} & \frac{1}{10k}+\frac{1}{2k}+\frac{1}{4k}+\frac{1}{5k} \end{bmatrix} \begin{bmatrix} V_{in} \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

The solution for this equation yields

v=[2365.4   1634.6   1826.9]T,   Vb=**v**(2)-**v**(3)=-192.3, Ia=**v**(2)/2000= 0.817   and   Ib=Vb/5000 = -0.038. These results are interpreted directly as

$$R_{in} = \frac{V_{in}}{I_{in}} = 2{,}365.4\,\Omega; \frac{V_a}{I_{in}} = 1{,}634.6\,\Omega; \qquad \frac{V_b}{I_{in}} = -192.3\ \Omega;$$

$$\frac{I_a}{I_{in}} = 0.817; \qquad \frac{I_b}{I_{in}} = -0.0138.$$

These values are now divided by Vin=**v**(1), already available from the resulting vector, to yield

$$\frac{V_a}{V_{in}} = \frac{1634.6}{2365.4} = 0.691, \qquad \frac{V_b}{V_{in}} = \frac{-192.3}{2365.4} = -0.0813,$$

$$\frac{I_a}{V_{in}} = \frac{0.817}{2365.4}S = 345.5\ \mu S, \qquad \frac{I_b}{V_{in}} = \frac{-0.0138}{2365.4}S = -16.26\ \mu S$$

End of Example

The mathematics of this example is more traditional, but the emphasis on the appropriate interpretation and use of results after (9) illustrates the point. Moreover, this example will serve easily for introducing the next two subsections.

III. Thevenin equivalent circuit

Thevenin's theorem states that any linear sub circuit is equivalent to a voltage source *Vth* in series with a resistance *Rth*. The voltage source is the open circuit voltage between the terminals and the resistance is the equivalent resistance seen at the port. This interpretation is the commonly used for finding the equivalent representation, and it is advised to continue doing it for simple circuits or for pedagogical reasons. However, when the circuit becomes complex or time is important, setting up the equations in the form (4) can be of help.

For that purpose, we add a dummy symbolical current source Itest at the port, as shown in Fig. 4a. When Itest=0, the source behaves as an open circuit, so the voltage across it will be Vth, (Fig. 4b). When all the sources in the port are set to zero, the source will see the equivalent resistance (Fig. 4c) so V_{AB}=RthItest. Following (8) and interpretation (9), we can therefore say that the solution for V_{AB} in Fig. 4a will be

$$V_{AB} = V_{th} + R_{th}I_{test} \tag{14}$$
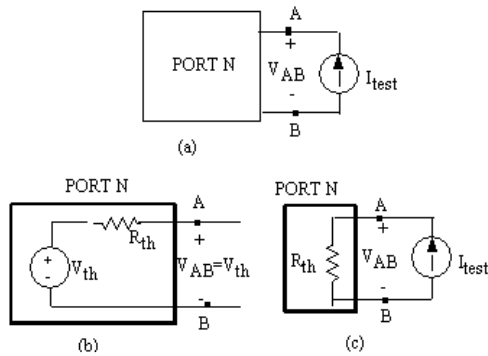
(a)

(b)   (c)

FIGURE 4 SETTINGS FOR FINDING THEVENIN'S EQUIVALENT
CIRCUIT USING A CURRENT SOURCE.

Recall that *Vth* in (14) is not necessarily a number; it can be a symbolic value itself or a combination as in example 1 or example 2. To enhance the interest of this remark, consider the following example.

Example 4. Find the Thevenin equivalent for the port of Fig. 5, where the independent sources have symbolic values.

The settings for the nodal equation are given in Fig. 6. The matrix equation for this circuit is

$$
\begin{bmatrix}
\frac{1}{1000}+\frac{1}{10000} & -\frac{1}{1E3} & 0 \\
-\frac{21}{1000} & \frac{21}{1000}+\frac{1}{5000}+\frac{1}{10000}+\frac{1}{2000} & -\frac{1}{2000} \\
0 & -\frac{1}{2000} & \frac{1}{300}+\frac{1}{500}+\frac{1}{2000}
\end{bmatrix}
\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} =
$$

$$
\begin{bmatrix}
\frac{1}{10000} & 0 & 1 \\
\frac{1}{10000} & 0 & 0 \\
\frac{1}{300} & 1 & 0
\end{bmatrix}
\begin{bmatrix} E_1 \\ I_s \\ I_{test} \end{bmatrix}
$$

The application of the numerical operation yields

$$
\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} =
\begin{bmatrix}
0.873 & 29.225 & 7418.412 \\
0.861 & 32.148 & 7160.253 \\
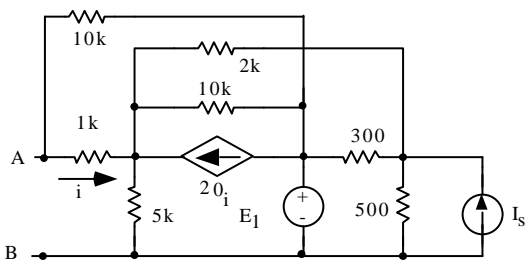0.645 & 174.184 & 613.736
\end{bmatrix}
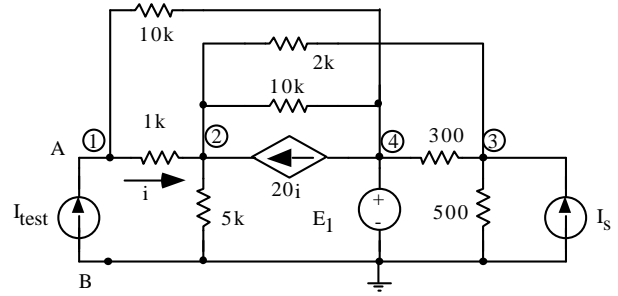$$

FIGURE 5   CIRCUIT FOR EXAMPLE 4

FIGURE 6 SETTINGS FOR CIRCUIT OF FIG. 5

To read the required result, remember that the first row corresponds to the voltage in the test source. Using (14) as reference, the column for I_{test}, the third one, yields the Thevenin resistance, while the rest of the columns yield the Thevenin voltage. Associating the numbers to the symbolic values as it was done in example 1, we have

$$V_{th} = 0.873\ E_1 + 29.225\ I_s; \quad R_{th} = 7.418\ k\Omega$$

End of Example

The Norton equivalent representation can be similarly worked out after the proper setting or the proper equations are considered.

IV. Two Ports.

A two port is a subcircuit where four or three terminals are identified for connection to the outside world as shown in Fig. 7 The variables identified with the two port are the voltages and currents V_1, I_1, V_2 and I_2.
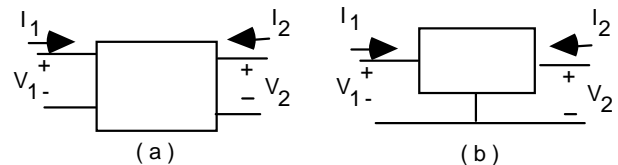

(a)   (b)

FIGURE 7 TWO PORTS a) FOUR TERMINAL, b) THREE TERMINAL

A two port without independent sources may be characterized by a set of two port parameters [10]. To find a given set of parameters, we exploit here the fact that in (4) the variables z_i are any type of parameter of interest. With this in mind, the procedure can be described as follows:

• Step 1: In each port introduce a symbolic source of any type, with voltage and current directions in the same direction as the corresponding port variables..

• Step 2: Set up the equations as usual, including all the port variables (hence, in nodal equations, for example, do not forget to include the current in the voltage source; in loop equations, the voltage of a current source).

• Step 3: If necessary, manipulate the equations so the "independent" variables appear in the right hand side.

• Step 4: Solve the equations. The submatrix formed with the rows of the "dependent" port magnitudes define the port parameters.

Let us illustrate with an example for the ABCD parameters, in which the equations are of the form

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \qquad (15)$$

with independent variables V_2 and $-I_2$ (note the sign)

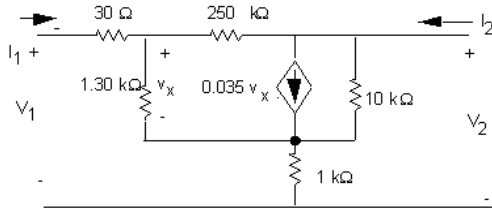**Example 5**: Find the ABCD parameters for the two port of Fig. 8.

FIG. 8 TWO PORT FOR EXAMPLE 5

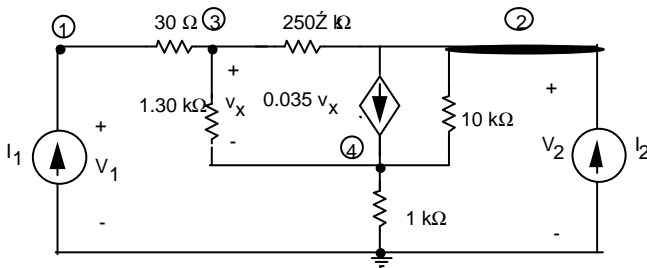The setting for nodal analysis is shown in Fig. 9

FIG. 9 TWO PORT PREPARED FOR NODAL ANALYSIS

The nodal equations, after manipulation to show the variables V2 and –I2 in the "known" side, become of the form $\mathbf{Ax=Bz}$, with vectors $\mathbf{x}=[V_1, I_1, V_3, V_4]^T$, $\mathbf{z}=[V_2, -I_2]^T$ and matrices

$$\mathbf{A} = \begin{bmatrix} \dfrac{1}{30} & -1 & -\dfrac{1}{30} & 0 \\[2mm] 0 & 0 & -\dfrac{1}{250E3}+0.035 & -\dfrac{1}{10E3}-0.035 \\[2mm] -\dfrac{1}{30} & 0 & \dfrac{1}{30}+\dfrac{1}{1300}+\dfrac{1}{250E3} & -\dfrac{1}{1300} \\[2mm] 0 & 0 & -\dfrac{1}{1300}-0.035 & \dfrac{1}{1000}+\dfrac{1}{1300}+\dfrac{1}{10E3}+0.035 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 0 & 0 \\[2mm] -\left(\dfrac{1}{10E3}+\dfrac{1}{250E3}\right) & -1 \\[2mm] \dfrac{1}{250E3} & 0 \\[2mm] \dfrac{1}{10E3} & 0 \end{bmatrix}$$

Doing the operation $\mathbf{A^{-1}B}$ we obtain the result

$$\begin{bmatrix} V_1 \\ I_1 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} -9.52E-3 & -1.06E3 \\ -6.34E-6 & -28.57E-3 \\ -9.33E-3 & -1.06E-3 \\ -6.34E-3 & -1.03E-3 \end{bmatrix} \begin{bmatrix} V_2 \\ -2 \end{bmatrix}$$

from which we interpret

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} -9.52E-3 & -1.06E3 \\ -6.34E-6 & -28.57E-3 \end{bmatrix}$$

End of Example

CONCLUSIONS

A new approach has been adopted to work with the equations naturally arising in linear circuits, in which the right hand side is usually a linear combination of sources or variables. With this approach, several advantages are obtained, among which we can mention the easiness to work numerical procedures where symbolic ones were necessary before. This feature is useful for students, both theoretically and practically, especially when no symbolic tools are available. This feature alone makes it attractive for use in early stages, when numerical complexities are of main concern.

Other advantages include the development of skills such as interpretation of computer (in this case, calculator) results, and knowing how equations relate directly to objectives and to properties of circuits.

The approach also allows the development of straight algorithms for the solution of common applications. Space limitations have not allowed a complete presentation of many more possibilities, which are left further discussion.

REFERENCES

[1] Pipes, L. A,. 'Solution of Variable Circuits by Matrices", *J. of Franklin Institute*, Vol. 224, 1937, pp. 766-777.

[2] Pipes, L. A., "The Matrix Theory of Four-Terminal Networks", *Phil. Mag., Series 7,* Vol 30, Nov. 1940, pp. 370-395.

[3] Balabanian, N and Bickart, T. *Linear Network Theory*, Matrix, OH: Chesterland, 1981.

[4] Moschytz, G., S., *Linear Integrated Networks: Fundamentals*, Van Nostrand Reinhold Co, New York, 1974

[5] Vlach, J., and Singhal K., *Computer Methods for Circuit Analysis and Design*, Van Nostrand Reinhold, New York, 1983

[6] Nilsson, J., Riedel, S. A., Riedel, D., *Electric Circuits*, Prentice Hall, NJ: Englewood Cliffs, 2004

[7] Madhu, S., *Linear Circuit Analysis,* Prentice Hall, NJ: Englewood Cliffs, 1988

[8] Irwin, D. J., Nelms R. M., *Basic Engineering Circuit Analysis*, 8th Ed. John Wiley and Sons, NJ, 2005

[9] AGottling J. G., *Matrix Analysis of Circuits Using Matlab*, Prentice Hall, NJ: Englewood Cliffs, 1995

[10] Palomera-Garcia, R. "Multipole and Multiport Analysis", in *Wiley Encyclopedia of Electrical and Electronics Engineering*, John Wiley & Sons, Vol. 14, 1999, pp 1-25

[11] http://www.abet.org/