A problem-based approach to Optimization: applied cases, from basic cycles to Engineering education

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Abstract **- Some Optimization principles applications are described, specifically aimed at students from basic cycles up to Engineering graduation degrees, through practical cases. Illustrative examples are developed, in a way both simple and instructive. For pre-university levels, the significance of scarce resources, optimality, and marginal values is illustrated by a simple (integer) Linear Programming (LP) case, to be presented at the municipal schools activities of ,"Expo-Escolas". A problem with limited dimension is adequate, with the calculations simplified to the proper level. The LP modelling framework has been recently included in the secondary cycle mathematical curriculum, so the students have the opportunity at "Expo-Escolas" to recognize the empowering from solving a complex problem. For Engineering graduation students, the focus is on the basic concepts behind LP methodology, its limitations and trade-offs achieved. For 2.nd cycle graduation (M. Sc.), a case study of higher level of complexity (Economic Lot Sizing Problem modelled by Integer Programming) is presented, with straightforward explanations, oriented to practice, and its reformulations are treated. For 3.rd cycle graduation (Ph. D.), the meaning and implications of uncertainty in mathematical solutions are interpreted, leading to an improvement of the robustness of the models and solutions.**

Index Terms - Engineering education; Problem-Based Learning; Optimization; Mixed Integer Linear Programming; heuristics.

INTRODUCTION

This communication describes several applications of Optimization principles, illustrating numerical examples in a way that is simultaneously simple and instructive. Those problem cases are aimed at students of different levels, from basic and secondary cycles to Engineering graduation degrees. Due to space economy, the problems are presented only implicitly.

Nowadays, the interest of Optimization in a risk environment is crucial, featuring probabilistic information, like the one associated to the obsolescence in personal computer industry, or in the allocation of electric power units to satisfy the fluctuating consumer demand, or also considering the long range planning of investments in the chemical industry. Usually, this type of problem situations also features the integrality of the decision variables, namely, modeling through binary variables.

A contrasting trend facing this increasing complexity of problem situations is the diminishing quality in basic sciences (Mathematics, Physics, Chemistry), or in maternal language competences, generally shown by the students entering the Engineering graduation courses. One approach that promotes the basic knowledge in science, the achievement of solutions to problem situations, and also allows the motivation of the students with best grades to choose the Engineering professional area is the one that conjugates problem-based learning with basic discrete mathematics. Thus, this approach is developed here, addressing optimization problems to different pre-university cycles (from first basic cycle to secondary cycle), and to the cycles of Bologna Engineering graduation (Graduate, M. Sc., PhD.).

For the basic cycles (first: 6 to 9 years old, up to 4 years of school; second: 10 and 11 years old, up to 6 years of school; third: 12 to 14 years old, up to 9 years of school), the significance of scarce resources and marginal values is illustrated by a simple (integer) Linear Programming (LP) case, built specifically to be presented at the municipal schools activities, the so-called "Expo-Escolas". A problem with limited dimension [1] is adequate, with the calculations simplified to the proper level, with the complexity growing from the basic cycles to the secondary cycle (15 to 17 years old, up to 12 years of school). For this secondary cycle a more complex case is presented, that relies on the importance of computing: the case dimension and the resolution time are out of hand, so the LP formulation is settled and a simple greedy algorithm is applied. It must be noted that the LP modelling framework has been included in the secondary cycle mathematical curriculum, so the students have the opportunity at "Expo-Escolas" to recognize the empowering represented by solving a complex problem, i.e., to obtain the numerical solution of the problem under study.

For Engineering graduation students, or even management science areas, the focus is on the basic concepts behind the methodology of (integer) Linear Programming, its limitations and trade-offs achieved. For 2.nd cycle graduation (M. Sc.), numerical instances of one case of high level of complexity (Integer Programming illustrated by the Economic Lot Sizing Problem) are treated, considering straightforward explanations that are practice-oriented, and alternative problem reformulations are developed. For 3.rd cycle graduation (Ph. D.), the implications of uncertainty in the mathematical solutions are interpreted, leading to an improvement of the robustness, both in models and in solutions, through a Stochastic Programming framework.

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The structure of this paper concerns: a review of the main issues to learn Optimization and addressing Engineering education, through Problem-Based Learning; then, a description of several problem-based approaches for pre-university studies; a presentation of some methodologies aimed at Engineering graduation Bologna cycles, by optimizing complex and open situations based on Economic Lot Sizing Problem; and finally, the main conclusions obtained.

LEARNING OPTIMIZATION ONTO ENGINEERING EDUCATION

An engineer deals with the science and economic principles in a manner to achieve the best possible solution for a problem situation, i.e., to optimize it. The competences needed to ensure the effectiveness on problem solving should be developed from the initial education levels, and should be deepened in graduation Engineering courses. One powerful tool in Optimization is the well-known Linear Programming modelling framework, which has been generalized to take into consideration the integrality of the decision variables and the uncertainty in the model parameters. A methodology usually applied to these complex, difficult and large dimension Optimization problems that cannot be efficiently and exactly solved, is through approximation procedures or heuristics.

I. Engineering education

There are some issues in training and educating Engineering professionals that must be taken into account. A significant fraction of the best students of the secondary cycle who achieved high grades in science disciplines (Mathematics, Physics, Chemistry), are often seduced by other professional areas, like Medicine. Other students that choose Engineering present some difficulties in sciences knowledge, and this will cause other quality losses in the learning process. Namely, absenteeism to theoretic classes has been generalized, and the scientific terminology tends to be neglected. Another known symptom is the negligence or lack of attention devoted to basic science disciplines, when compared with the complementary or speciality ones.

When pre-university students recognize the complexity and uncertainty of knowledge (Pluto is no more a planet, thus a poll is peremptory and absolute; if zero is not a natural counting number, we have no way to count the number of elements of an empty set; the evolution of atomic models, of the solar system ideas, even of the light concepts, etc.), the central work of a science education can rely on modelling, linking theory and critical arguments considering some basic ideas (axioms); developing proof procedures within logical reasoning; and to compare and operate with alternative systems of thought.

At this point, we must refer that there is a perspective [2] that intends to improve some discrete mathematics topics in undergraduate curricula (graph vertex-edge theory, combinatorial and counting methods, proof procedures, relations and algorithms), with bivalence properties that provides: ***i)*** necessary and interesting mathematical topics; ***ii)*** new strategies to motivate the students in the study of mathematics, revitalizing it.

The competences that one today needs are related to thinking critically, to solve problems, and to make decisions. To accomplish this with effectiveness, one must use reasoning and strategies, in contrast with the usual competence required in the basic operations, when students are trained to directly apply rules and mathematic formulas. The goal is now to motivate the students to: discuss and apply the methods to solve complex and problematic questions; understand and use concepts and techniques from different areas of science and mathematics; and build solving strategies.

In the first year of Engineering education it is usual to verify the weakness of students on disciplines of mathematics or physics, where the lack in science basics or in mathematical techniques are two issues. But one serious issue is the non-understanding of problem solving processes, namely, when a sequence of interrelated steps is needed. In fact, many students coming to university had no training at all in the process of solving real problems: usually, examinations presented single step questions that test the knowledge of individual principles. When pre-university students need to take multi-step methods, and if they face a large set of information, usually they will not know the method to choose. The goal in promoting complex problems with incomplete information on an open framework contrasts with structured problems that can be worked out with certainty, based on procedures or techniques developed by repetition.

So, the basic skills on problem solving that Engineering students need should be developed in school, with Engineering curricula built on them. In an attempt to resolve the lack of those skills, there are some trends in Engineering education claiming that problem-solving competences should be systematically developed in the first year of the Engineering curricula.

II. Learning with problems

Problem-based learning was initiated at McMaster University Medical School and this learning process has been implemented in undergraduate and graduate curricula all around the world. It emulates the approaches used to solve problem situations encountered in real and professional life. The curricula in problem-based learning consist of selected and designed problems that require from the student the acquisition of: critical knowledge; problem-solving competences; learning and constructive strategies; and team participation skills.

Typically, the problem cases for studying are written objectively, including an overview of the situation, its context, and the major decisions that must be made. Instead of leading the students to one right answer, it is intended to develop the students reasoning, to frame the problem situation, to generate analytical or numerical solutions, and so the strategy built this way can be applied to other situations. Also, the features of uncertainty and complexity that characterize problem-solving approach~~,~~ are incorporating bi-level benefits: the uncertainty is contrasting with the unchanged and absolute knowledge; the methods applied to a complex problem are driving to strategic processes, comparative or evaluative analysis, and perspective and criticism upon results.

Facing a problem situation, the tutor is not necessarily an expert or specialist in the theme, but a motivator of the learning experience for the student. Also, the specific problem should be designed to emphasize aspects of basic sciences, promoting an additional level of cognition, in the sphere of real application of the knowledge from those basic areas.

III. Optimization

Optimization constitutes an essential issue of the contemporary world, with the desired sustainable growth, with finite resources promoting a stronger integration of economic activities and a larger complexity of the problem situations to be treated. In this context, it must apply some quantitative mathematical techniques, not only to model the problem situation, but also to simulate the real system behaviour, or also to optimize the decisions.

Optimizing through Mathematical Programming, namely using Mixed Integer Linear Programming conjugates the integrality specification of some decision variables and the formulation flexibility derived from binary variables, with the efficiency of Linear Programming methods. Also, one can associate a logic nature of affirmative or negative decision, according to the optimum value of each binary variable. Then, one can build complex logic propositions by conjugating these binary variables, or just formulating multi-criteria costs like those considering fixed charge components, formulating the usual economy of scale [3].

Due to the interest of some engineering problems formulated recurring to integer or binary variables for several specialities (chemical, electrical, computing, etc.), these problems often constitute NP-complete or NP-hard (complexity computational class) problems [4]-[5]. A usual approach when facing a large sized problem of the referred classes is just to obtain a set of admissible and near optimum solutions, by applying heuristics or approximation algorithms that can be processed in polynomial time.

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Engineering education should, thus, be directed to solve problematic and complex situations, optimizing them by considering either the best solution or also good solutions achievable in a acceptable amount of time. One possible approach is to use approximation procedures, and all the competences needed for exact or heuristic methods can be developed by problem-based learning that combines characteristics both from the application goal and the means used.

PROBLEM-BASED APPROACHES FOR PRE- UNIVERSITY STUDIES

In science areas, like Mathematics, Physics or Chemistry, problem-based approaches constitute a universal learning framework, within which defying situations are presented to the students, who can use several strategies and resolution methods. It should outperform the usual exercises application, where one applies a simple algorithm or formula, in a repetitive or mechanical manner, just to find a closed solution.

There is some educational research that shows that we learn better when we do some activity connected to the desired goal, with adequate duration and intensity. The issue is how to select the type of activity the student should follow to achieve his purpose, so that he may reach the objectives and goals defined. Another issue is to empower the student to initiate mathematical explorations and to build what he will need to develop a plan to find answers.

The paradigm to problem-based learning involves the following simplified and iterative procedure: ***i)*** introducing an open-ended problem or scenario; ***ii)*** listing the data from the situation, as well as refreshing the related knowledge; ***iii)*** developing a specific problem statement; ***iv)*** listing the information, quantitative or qualitative, needed and that has to be found; ***v)*** listing actions to be taken, formulating and testing alternative hypotheses within interrelated and sequential actions; ***vi)*** presenting and supporting the solution(s) proposed.

Also, the number and variety of problems in discrete mathematics [2] have a suitable level of difficulty, because they are friendly (simply stated, then easy to understand) and motivating (challenging to solve, but the instances presented can be solved). Thus, it is motivating for the student to learn the mathematics content, both in discrete mathematics and in traditional areas, like numbers, algebra and geometry, which are closely related to discrete mathematics. It must be noted that we should consider the mathematical experience of the students, early at the problem design stage.

I. Learning Optimization at the basic cycles

The significance of scarce resources and their marginal values are problematized in a way to fit the basic school cycle. The problem situation is illustrated by a simple Linear Programming case that features integer decision variables, and is built specifically to be presented at the municipal schools activities of “Expo-Escolas”. It consists both of an open-ended scenario and the consideration of “What if?” analysis, where the students from different degrees (first, second or third cycle) of knowledge are suitably challenged. The partial questions are sequenced in a manner that permits the students to make their own resolution methods, performing an autonomous approach, but within the team work (two or three elements for each group) and the suitable tutorial supervision, so that the goals aimed at each cycle can be accomplished. These goals are not defined in a tight partition format, but they allow the students to build self-contained strategies to solve the problem issues. Thus, these can overlap the first and second cycles, or the second and third cycles, and the possibility is not out of sight for a student of the first cycle to reach some goals aimed at the third cycle.

Specifically to the first and second cycles, a problem with limited dimension [1] is adequate, with the calculations simplified to the proper level: they only include integer non-negative numbers, and the sums and subtractions do not reach 20.

Thus, the teaching objectives consider simple combinatorial counting, the basic algebraic operations (addition, subtraction, multiplication, division), and comparing several alternative solutions. The competences laboured are related to the numeric natural system and the four operations algorithms and their properties to make the calculations easiest. The framework of proportionality is intended preferentially to second cycle students (the data can be proportionally enlarged), and operations with fractional numbers can be introduced here. Also, the comparative reasoning can be stimulated to recognize that the optimum solution has been achieved.

As regards problem analysis, the students are stimulated to discuss among the group to identify the specific quantitative problem, and thus to take decisions about the methods and activities directed to obtain the optimum solution. So, the students should: select the relevant information; adopt the strategy and plan the methodology that suits the group better; and assess and adjust these strategies and methods to the desired goals. Just to promote responsibility, autonomy and self-initiative of the students, the tutorial approach should be "surgical", meaning that the supervisor accounts the team work and briefly introduces questions that could lead to a correct approach. This way, the mathematic main goals of the first and second cycles can be accomplished.

The problem situation is not closed, by introducing simple variations on the case data, promoting a sensitivity or "What If?" analysis. These issues can be understood as extension questions, asked after the students perform correctly the reasoning about: the identification of optimum solution; and the estimation of marginal values. Consequently, there is not a defined line between the knowledge or competences of successive basic cycles, the issues of optimality conditions and marginal values being the main principles where the third cycle students will perform the sensitivity analysis.

For the third cycle, we wish to enlarge the problem dimension in a proportional way (and afterwards, if possible, a non-proportional one) and this assumes that the conditions conducing to optimum solution and marginal values are well understood by these students.

The teaching objectives consider either simple statistic principles, or algebraic and functions manipulation. The competences related to the first objective are the research and treatment of quantitative data, and the realization of simple combinatorial counting. With the former objective, we consider the identification of proportionality situations, the comprehension and interpretation of the associated mathematic formulas, the constitution and verification of a simple linear system, and the realization of simple algebraic procedures. The situation also allows a better discernment of the function concept, and with the recourse to a personal computer, the students can build the corresponding tables and graphics.

On performing the problem analysis, the students deepen the quantitative approach and the self-assessment of the strategy and methodology applied, and they further develop the capacity to work inside the group. The supervisor maintains the noninterference status, to promote the creativity and self-contained developing processes, but the tutorial approach should provide: some clues on the students approximation procedures; some alternative and non-optimal solutions, for verification of the equations; pertinent indications on manipulating the personal computer, to promote the correct elaboration of graphics and tables.

The problem case is open-ended, in the sense that we introduce some data variability that transforms the proportional instance of the problem into non-proportional. Even in the latter instance, an approximation procedure can lead to the optimum solution, and the students should be aware of this fact. Another possible extension is introducing the LP formulation and the corresponding graphic representation, which is treated in secondary cycle.

II. Learning Optimization at the secondary cycle

In the secondary cycle, a large and non-proportional instance of the problem situation is presented, so its dimension and resolution time are out of hand. Thus, the students must appeal to LP formulation and perform their calculations with computer support. The LP modeling framework has been recently included in the mathematic curriculum of this cycle, so we have the opportunity at "Expo-Escolas" to empower the secondary students with a tool to solve LP problems.

The main teaching objectives are the mathematical modelling through LP formulation, the graphic construction of the linear functions using straight lines and their interceptions, the study of variations through derivative concepts, and the potential development of an approximation (greedy) algorithm.

Analysing the problematic situation, each group of students are induced to model it using the LP framework, within the associated simplifications and approximations, and thus building a satisfactory mathematical representation of the problem.

The supervisor role consists in supporting the utilization of tutorial free software, available through the World Wide Web, which provides the optimum solution of the problem under study and its graphical approach. The problem instance is numerically solved and the importance of computing is enhanced. For each group LP formulation settled, the computational solution is provided; also, students are defied to build a simple greedy algorithm and to verify the quality of their numerical solutions; in parallel, they are also challenged to perform a graphical resolution of the two decision variable problem. Finally, students should be aware that both the solutions obtained and the conclusion derived from their model must be validated in the real environment.

Some potential extensions include the introduction of some topics usually performed at graduation level: the reasoning of the well-known Simplex method; interpreting dual values; and performing sensitivity analysis.

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Some problem situations are aimed at students of pre-university cycles, which challenge them to use alternative strategies and resolution methods, instead of the usual mechanized exercise. The students develop proper activities connected to the objectives of their knowledge level. Those activities are designed considering the mathematical experience of the students, who thus are empowered to pursue mathematical explorations.

PROBLEM BASED OPTIMIZATION AIMED TO ENGINEERING GRADUATION BOLOGNA CYCLES

The main goals of Engineering education aim to scientific and speculative spirit, educating and training the students for professional engineering areas, promoting scientific or applied research, considering a global or European environment, relating with the community in general, for example, communicating and spreading these specific knowledge.

An obstacle faced by who designs Bologna curricula is the process how to determine the competences desired, plan the content of the course, and to connect goals and objectives to the measures of performance, i.e., to measure the criteria defined for the grades. The Bologna principle of the student's centered learning process implicitly involves problem-based learning, where the student assumes increasing responsibility, with improved motivation and with strategies built for reasoning in face of new problematic situations that will be presented all life long. Another common aspect relays in the resources utilization, like tutorial monitoring, with continuous supervision and evaluation, guiding the activities made by students to solve the problems.

The paradigm of Engineering problem solving [6]-[8] generally involves: a problematic situation formulated through a specific word problem; this specific problem is translated into mathematical formulation; the resulting mathematic model is solved, analytically or numerically; the answer is analysed and stated. However, this paradigm of problem solving presupposes that a engineer always know the resolution route, forgotten that he has to travel over the path. And this travel involves a complex relationship between reasoning and feeling, often involves bad partial decisions, and some negative results can open the way to positive ones, accomplishing with good and possible solutions to the problematic situation. Thus, the competences on solving problems are conjugated with the basic knowledge above they must be built.

I. Learning Optimization at Engineering graduation cycle

For Engineering graduation students or Management Science areas, the focus is on the basic concepts behind the methodology of (integer) Linear Programming: namely, LP limitations, featuring deterministic data and positive continuous decision variables, and the trade-offs achieved, specifically from the optimal activity analysis under scarce resources.

The situation problematized is directed to the comprehension and resolution of an LP multiperiod ("dynamic") problem, and through developing applied research, it achieves some progresses on critical analysis and innovation competences that will be essential in the engineering professional activities. Namely, the formulation of the objective function and restrictions are carefully detailed, the quality and the number of data are worked, several alternative (even conflicting) models are compared, and the computational implementation, numerical resolution and perspective on results are considered.

The main objectives are related to the competences and knowledge deepened in the Optimization theme through LP formulation, with higher and peak levels of knowledge, their proper and professional applications to the Engineering speciality, and assessing, reasoning and building in an autonomous way that could be generalized in all the professional career.

Possible extensions are related to other modelling types of Optimization, like Non-Linear Programming and related issues (Mixed Integer Non-Linear programming, Global Optimization), but the following characteristics of LP are pertinent: the numerical robustness and efficiency of the Simplex method, straightforward for large dimension problems; the effectiveness of duality and the studies of sensitivity analysis; the usual generalizations to Parametric LP, Stochastic LP or even to Integer LP. The referred extensions are directly caused by reasoning on an applied context, through technical assessment, physical limitations or due to logical relations between variables or system components. That way, some issues naturally treated on higher cycles are here introduced.

II. Learning Optimization in the Engineering master cycle

For 2.nd cycle graduation (M. Sc.), numerical instances of a case of high level of complexity (Economic Lot Sizing Problem) are presented, and problem reformulations are treated. The referred multiperiod problem is usually modelled through Integer Linear Programming, due to the occurring binary decision variables, and the Capacitated case constitutes an NP-hard one. However, the Uncapacitated case can be treated through several reformulations, dual procedures, or graph optimization like Shortest Path.

The main objectives consist of developing and deepening the graduation knowledge, in an innovation and research environment, in a manner applied to new, generalized and multidisciplinary problem situations occurring in the Engineering speciality, integrating complex issues where the information is incomplete or limited, and promoting autonomous and self-contained learning on the professional and research speciality.

Several extensions are on the public domain, like applying similar procedures to Location problems, Capacity Expansion problems, or using ELS sub-problems on production-inventory formulations. Even the uncertainty on data can be considered [9], leading to the Stochastic Programming framework, and the Two-Stage (and Multi-Stage) Stochastic Programming is introduced, attending to the promotion of robustness or also considering integer variables in stages after the first one.

III. Learning Optimization in the Engineering doctorate cycle

For 3.rd cycle graduation (Ph. D.), the meaning and implications of the mathematical solutions are interpreted, leading to an improvement of the robustness of the models and solutions. The problematized situation challenges Ph.D. students to promote scientific research based on instances of ELS problems, featuring probabilistic parameters or scenarios that demand stochastic treatment, and they develop the analysis and conception competences on the Robust

Optimization framework or in the treatment of stochastic integer variables. Thus, Ph.D. students are trained and educated on each specific Engineering area using the flexibility, both on formulation and the application theme, which characterizes the well-known ELS problem. The main issue is the comparative analysis of methods leading to a proper post-optimality study, namely due to the variability on data. This promotes the assessment and the risk treatment according to the relationships established among economic estimators and technical parameters.

Aimed at each specific scientific Engineering area, the objectives are the promotion of the competences on the systematization of the knowledge, the application of research methods, the conception and design of the academic and innovative research, and the assessment, analysis and development of new and complex concepts.

The issue described above is current and open-ended, and the extensions to other problems formulated using integer or binary variables are straightforward (like power Unit Commitment, chemical Process Synthesis, Travelling Salesman problems). Several approaches are prominent: graph or network reformulations, specific techniques on Integer Programming, on Global Optimization, aimed at large sized problems are a must. Thus, the systematization on heuristics [10] and approximation procedures development, and its conjugation with exact algorithms like the enumerative branch-and-bound, are also an issue in sight.

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A problem-learning approach to the Engineering Bologna cycles is presented: at the graduation cycle, the concepts characterizing the LP framework, its limitations and trade-offs achieved; at the master cycle, the numerical and implementation issues related to the efficient treatment suitable to integrality of decision variables; at the doctorate cycle, the interpretation of uncertainty on data requiring risk analysis and treatment, even conjugating the stochastic and the integrality issues. This problem-learning approach points to the proper computational treatment, obtaining optimum or satisfactory and realizable non-optimal solutions, and this way the Engineering professional does not become software-limited and can apply his competences autonomously, widely, under new and challenging contexts.

CONCLUSIONS

Several practical cases are described —only implicitly due to space economy—, specifically aimed at students from the basic cycles up to Engineering graduation degrees, and featuring some Optimization principles.

Engineering education should be directed to solve problematic and complex situations, optimizing them by considering either the best solution or good ones achievable in acceptable time. The competences needed for exact or heuristic methods can be developed by problem-based learning, which combines issues both from the application goal and the means used.

For pre-university levels, the significance of scarce resources, optimality, and marginal values is illustrated by a simple (integer) LP case, to be presented at the municipal schools activities of "Expo-Escolas". Those activities are challenging, defying the students to use alternative strategies and methods, and are connected to the objectives of their knowledge level. Those activities are designed considering the students' mathematical experience, who thus are empowered to pursue mathematical explorations.

For Engineering graduation students, the focus is on the basic concepts of the LP methodology, its limitations and trade-offs. For 2.nd cycle graduation (M. Sc.), a case study related to the Economic Lot Sizing Problem and modelled by Integer Programming is treated in a practice-oriented manner, and its reformulations are performed. For 3.rd cycle graduation (Ph. D.), the implications of uncertainty in the mathematical solutions are interpreted, leading to an improvement of the robustness of the models and solutions.

So, we present a problem-learning approach to Engineering education, also considering the pre-university cycles, and this approach is directed to develop competences in the Optimization area through LP. The importance of the integrality of decision variables and of data uncertainties is related to the empowerment of students achieving computer results and not being software-limited, through application of heuristic or approximation procedures adequate to the problem size, to the quality required for the solution and to the specific computational properties.

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