Contribution to postcritical non-linear behaviour of elastic plane trusses discretised by FEM - Comparison results obtained by „Exact methods“ with Newton-Raphson method.

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Abstract – **Department of Mechanics and Department of Mechanics of Materials of VŠB- TUO, Faculty of Mechanical Engineering supports study program "Applied Mechanics" for bachelor, master and doctoral study.**
One of the study part is "Computing Mechanics". It means applications of numerical methods and finite element methods in mechanics. Students of this specialisation study problem of linear buckling and the basement of the non-linear buckling. This paper compares the results of exact method with Newton-Raphson methods on simple example.

Index Terms : Students, FEM, buckling, linear and non-linear theory.

INTRODUCTION

Students of the specialisation Applied Mechanics study classical linear buckling theory. They study finite element method (FEM) too. They are able to use program ANSYS but it is necessary them to know the basement and "resources" of FEM – it means shape function, equation assembly etc. They study the basement of the non-linear buckling. It is very important to show them the result of non-linear buckling and result obtained by "Exact methods" for simple example and compare it. This access shows them the "possibility to math" (see example no. 1).
If the exact solution is not achievable (not so simple examples), it is possible to obtain equilibrium path by "modified" Arc-length methods. For this example, the new programme in MATLAB has been created (see example no.2).

LINEAR SOLUTION

In linear theory, the solution is solving using the equations

$$[[K]+\lambda\,[\mathrm{K_G}]]\cdot\{R_g\}=\{0\}$$

$$\det[[K]+\lambda\,[\mathrm{K_G}]]=0$$

The solution is critical finding coefficient λ The matrix is not depend of displacement.

NON-LINEAR SOLUTION

First step – shape function $\lfloor N_\zeta \rfloor$ for bar element. Bar element with coordinate system is on Fig. 1.

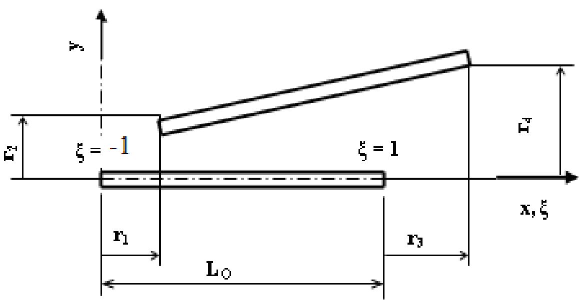


Fig. 1

The shape function $\lfloor N_\zeta \rfloor$ is

$$\{r_\xi\}=\begin{Bmatrix} r_{x\xi} \\ r_{y\xi}\end{Bmatrix}=\begin{bmatrix}\frac{1}{2}(1-\xi); & 0 & \frac{1}{2}(1+\xi); & 0\\ 0 & \frac{1}{2}(1-\xi); & 0 & \frac{1}{2}(1+\xi)\end{bmatrix}\begin{Bmatrix} r_1\\ r_2\\ r_3\\ r_4\end{Bmatrix}=\lfloor N_\xi \rfloor\{r\}$$

Potential energy of deformation U is:

$$U = \frac{1}{2}\{R\}^T \lfloor_u k \rfloor \{R\} \qquad (1)$$

The first derivation of potential energy U gives the secant stiffness matrix $\lfloor_p k \rfloor$ and internal load vector $\{b\}$

$$\frac{\partial U}{\partial\{R\}} = \left(\lfloor_u k \rfloor + \frac{1}{2}\{R\}^T \frac{\partial \lfloor_u k \rfloor}{\{R\}} \right)\{R\} = \lfloor_p k \rfloor \{R\} = \{b\} \qquad (2)$$

The derivation of internal load vector $\{b\}$ gives tangent stiffness matrix $\lfloor_t k \rfloor$

$$\frac{\partial\{b\}}{\partial\{R\}} = \left(\lfloor_p k \rfloor + \{R\}^T \frac{\partial \lfloor_p k \rfloor}{\{R\}} \right)\{R\} = \lfloor_t k \rfloor \qquad (3)$$

On the base of the valid Greens equations for strain, $\varepsilon_G = 0{,}04$ (large displacement but small strain) the problem was deduction of the all three types of non-linear stiffness matrixes ($\lfloor_u k \rfloor, \lfloor_t k \rfloor, \lfloor_p k \rfloor$) for plane link element in deformation variant of FEM - on the course of deduction **<u>not to neglect non-linear member</u>**, as it is standard in known methods.

The new type of the stiffness matrix $\lfloor_u k \rfloor$ consists from 4 part – 4 sub-matrix. The first one is constant, the second and third ones are dependent on displacement and the fourths one is function of displacement square.

$\lfloor_p k \rfloor$ is secant stiffness matrix, $\lfloor_t k \rfloor$ is tangent stiffness matrix and $\{b\}$ is internal load vector.

The solution was calculated in global coordinate system.

Postcritical non-linear behaviour of elastic plane trusses investigation (it means specification of critical points) was based on study of equilibrium path, determinant $\lfloor_t k \rfloor$ valuation and valuation of factor ψ (for specification of bifurcation points).

For the demonstration was chosen simple example. This example is possible to solve "Exact Methods".

RESULT

1. EXAMPLE

For the demonstration was chosen simple example (Fig. 2). This example is possible to solve "Exact Methods".

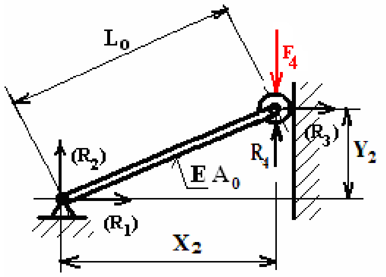
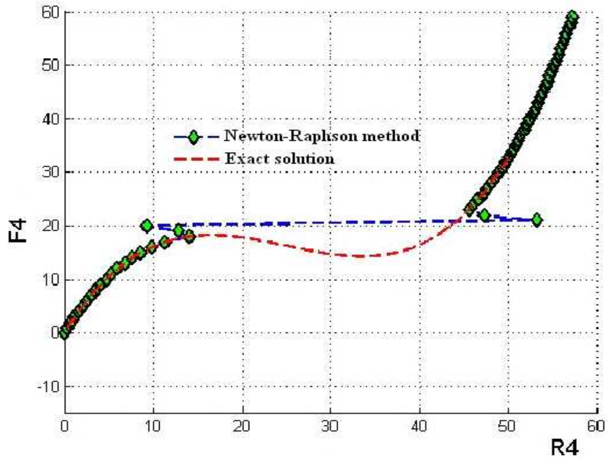


Fig. 2
X2 = 2500 mm, Y2 = 25 mm, E.Ao = 50 MN

The result obtained by "Exact methods" and by Newton-Raphson methods in shown in Fig. 3

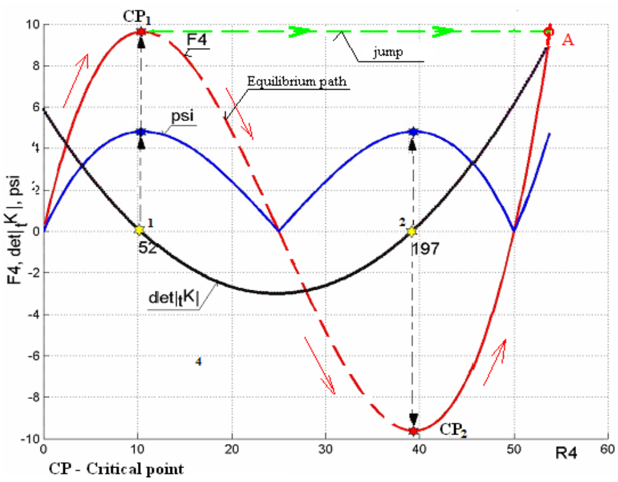
Fig. 3



The Newton-Raphson method is not right. It is not possible to detect critical point on the equilibrium path.

The results obtained on the base of the new matrix mentioned above, is shown on Fig. 4. This method gives better conception of equilibrium path and better conception of buckling of truss system.

Fig. 4



2. EXAMPLE

The example (2) shows the result of non-linear buckling for the example, where the "exact" result is not obtained (Fig. 5). Using "modified" Arc – Length method (**<u>not to neglect non-linear member)</u>**, in new "program" in MATLAB, it is possible to obtain equilibrium path with critical points.
For determination of the equilibrium path and critical points (bifurcation and limit points) it is necessary to know:

- The points, where the $\det\left[{}_tK\right]=0$ - It is critical point
- The points, where:

$$\psi = \{n\}^T\{F\} \quad \begin{cases} = 0 & \text{Bifurcation point} \\ \neq 0 & \text{Limit point} \end{cases}$$

ψ - Check parameter
$\{n\}$- Eigen value of $\left[{}_tK\right]$
$\{F\}$- Load vector

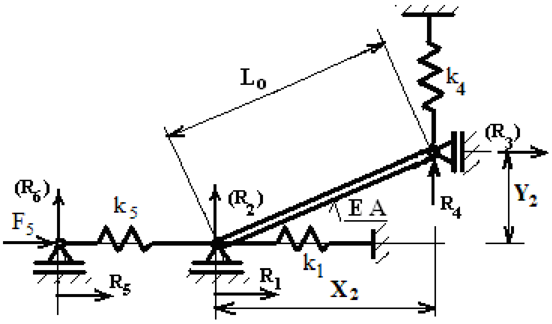


Fig.5 - Example 2.

The parameters of the system:
$X_2 = 2500$ mm, $Y_2 = 25$ mm, $E \cdot A_0 = 50$ MN
Spring constants **:** $\mathbf{k_1} = 0{,}25\ \text{N.mm}^{-1}$, $\mathbf{k_4} = 1{,}5\ \text{N.mm}^{-1}$,
$\mathbf{k_5} = 1\ \text{N.mm}^{-1}$

The system has 3 degree of freedom.

The course of force **F5**, determinant of tangent matrix $\det\left[{}_tK\right]=0$, check parameter ψ on displacement **R1, R2, R3** is shown on fig. 6, 7 and 8.
This system has on equilibrium path 2 critical points – 2 Limit points (**LB1**, **LB2**). Between this points, the system is unstable. The snap could appears between **LB1** and **LB2**

The system has not bifurcation points.

By use of Newton-Raphson method is not possible to obtain the "shape of equilibrium path". The calculation collapses close to critical point.

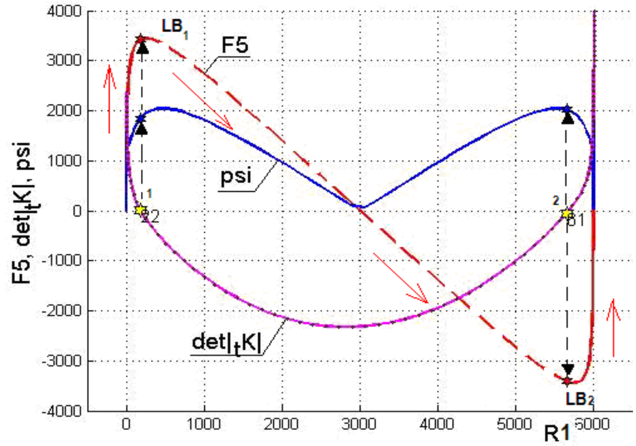


Fig. 7 – Course **F5**, $\det\left[{}_tK\right]=0$, ψ on **R1**

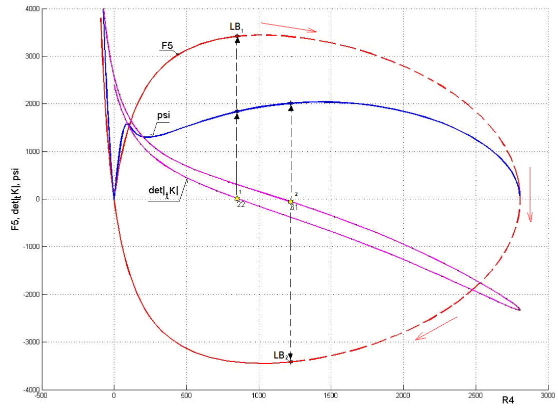


Fig. 8 – Course **F5**, $\det\left[{}_tK\right]=0$, ψ on **R4**

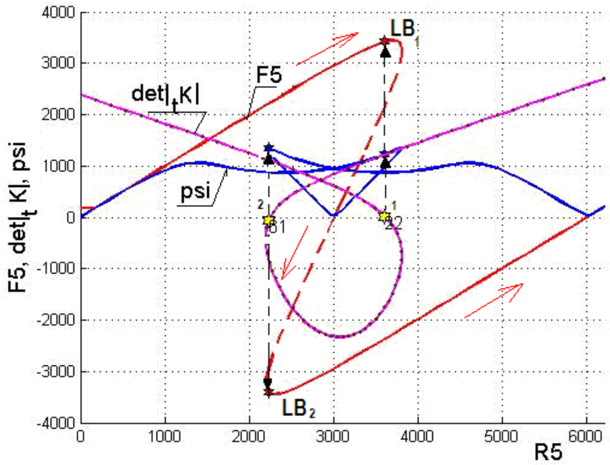


Fig. 9 – Course **F5**, $\det\left[{}_t K\right] = 0$, ψ on **R5**

CONCLUSION

These results clearly show the students the influence chosen type of method over “result” . It is very important to know the restriction of the methods. This examples and this method will be part of study materials for students of Applied Mechanics specialization.

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